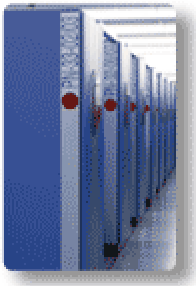

Nonlinear Force-Free Field Extrapolation

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Outlook

- Introduction
  - Why extrapolation
  - Why NLFF
- extrapolation methods (review)
- examples
- problems and issues

Why extrapolation?

- The 3D structure of magnetic field is crucially important to understand
  - the stability of active region
  - magnetic helicity & free magnetic energy
  - magnetic topology, current sheet and quasi-separatrix layer
  - direct comparison with theoretical models
- However, magnetogram can provide only the 2D data mainly on photospheric surface.

2D phospheric data should be extrapolated to 3D space.

Nonlinear Force-Free

- Plasma beta in the solar corona is usually much less than unity.
- Force-free field could be a good approximation for the magnetic field in low corona.

$$(\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p = 0$$

$$\nabla \times \mathbf{B} = \alpha(\mathbf{r})\mathbf{B}$$

$$\nabla \cdot \mathbf{B} = 0$$

⇩

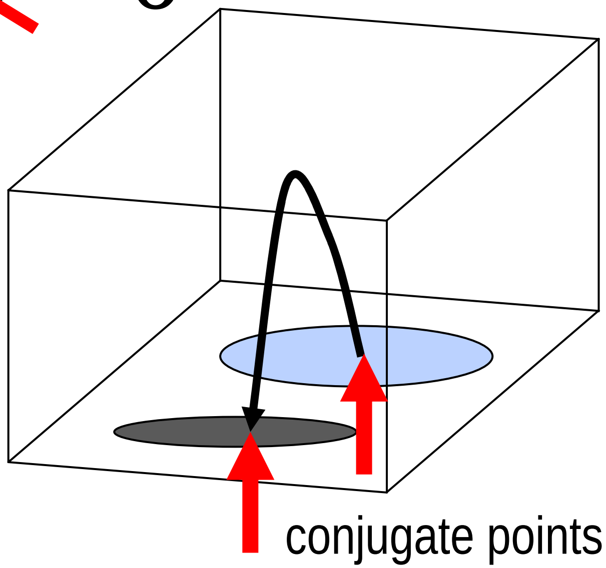
$$(\mathbf{B} \cdot \nabla)\alpha = 0$$

α is constant on each magnetic field line.

boundary condition

$$\alpha(\mathbf{r}) \subset \partial\Omega^{\pm}$$

$$Bz \subset \partial\Omega$$



conjugate points

NLFF Solvers

- Direct Integration
  (Wu 1990, Cuperman et al. 1991)
- Grad-Rubin-type methods
  (Sakurai 1981; Amari et al. 1997, 1999; Régnieret al. 2002; Wheatland 2004)
- Relaxation-type methods
  - stress & relax method (Roumeliotis 1996; McClymont, Jiao & Mikic; Valori et al. 2006)
  - optimization method
    (e.g. Wheatland et al. 2000, Wiegelmann & Neukirch 2003, McTiernan, ...)
- Boundary integral method
  (Yan & Sakurai 1997, 2000, Yan & Li 2006)

Grad-Rubin method

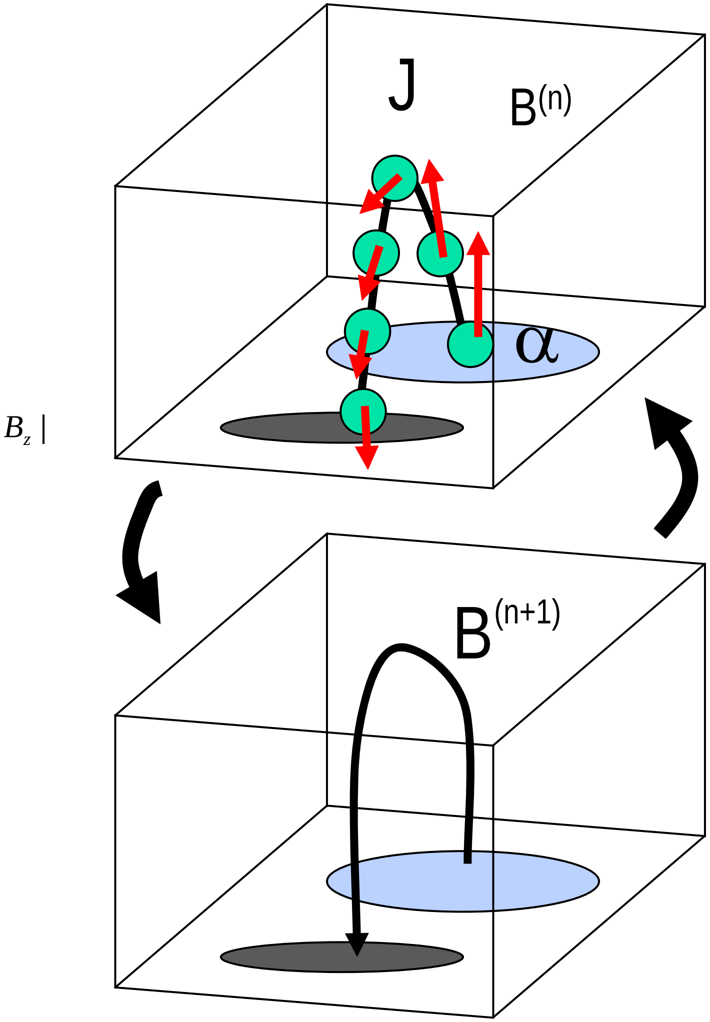
$$\alpha^{(n)}|_{\partial\Omega^+} = (\nabla\times\mathbf{B})_z / B_z$$

$$\mathbf{B}^{(n)}\cdot\nabla\alpha^{(n)} = 0$$

$$\nabla\times\mathbf{B}^{(n+1)} = \alpha^{(n)}\mathbf{B}^{(n)} \quad in \;\; \Omega$$

$$\nabla\cdot\mathbf{B}^{(n+1)} = 0$$

$$B_z^{(n+1)}|_{\partial\Omega} = B_z$$

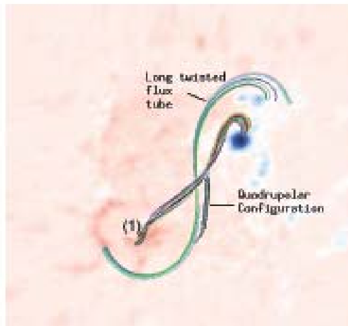


boundary conditions

$\alpha|_{\partial\Omega^+}$ $\quad B_z|_{\partial\Omega}$

fit to the force-free equation.

Regnier et al. 2002



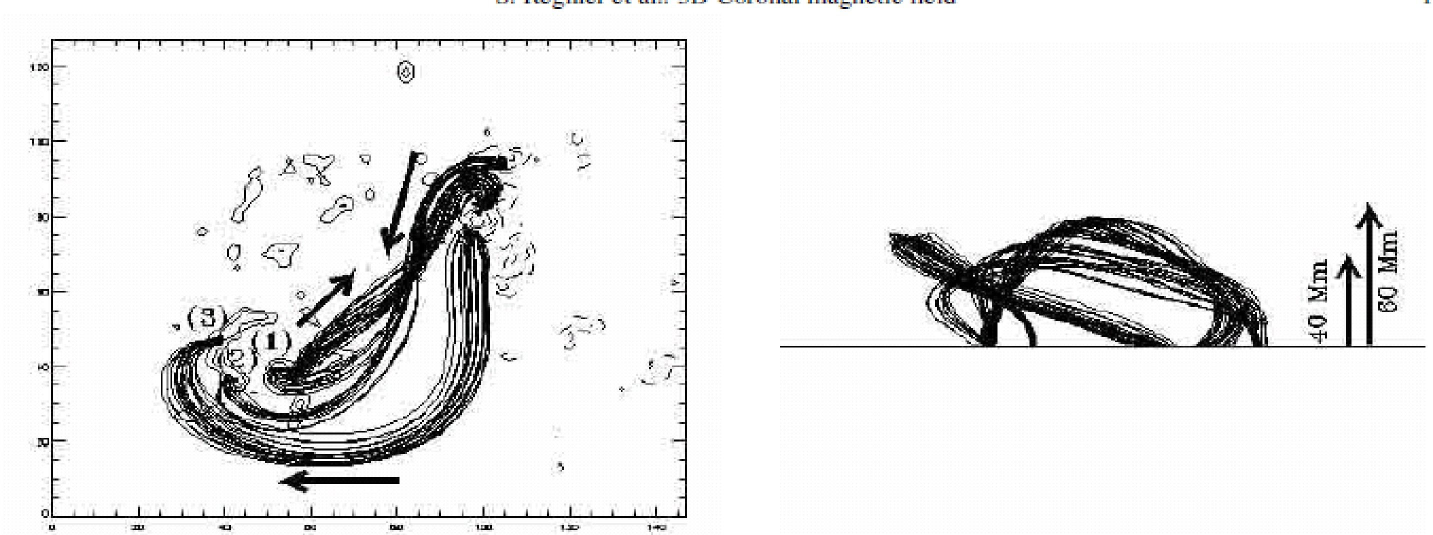


Fig. 9. Three characteristic magnetic flux tubes seen from top view (left) and from side view (right). The photospheric positive (resp. negative) polarity is drawn as solid (resp. dashed) contours. Black arrows (left) indicate the direction of the electric current density on each flux tube. The estimated height of flux tubes is indicated on the right image.

Relaxation Method

$$\rho \frac{d\mathbf{V}}{dt} = (\nabla \times \mathbf{B}) \times \mathbf{B} - \nu \mathbf{V} \quad \rightarrow \quad \mathbf{V} = \nu^{-1} (\nabla \times \mathbf{B}) \times \mathbf{B}$$

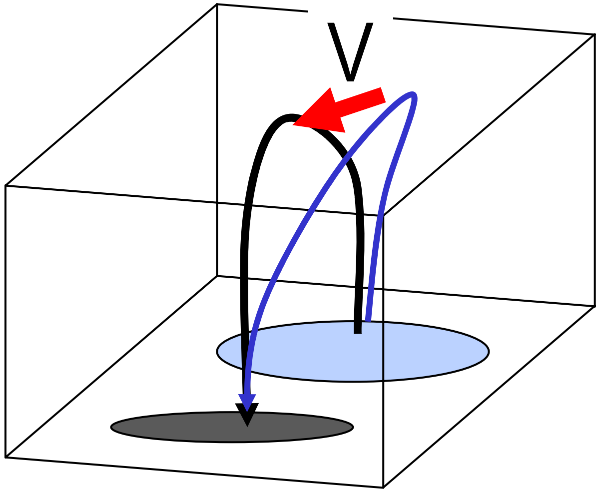
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

$$\eta = \eta(\mathbf{V})$$

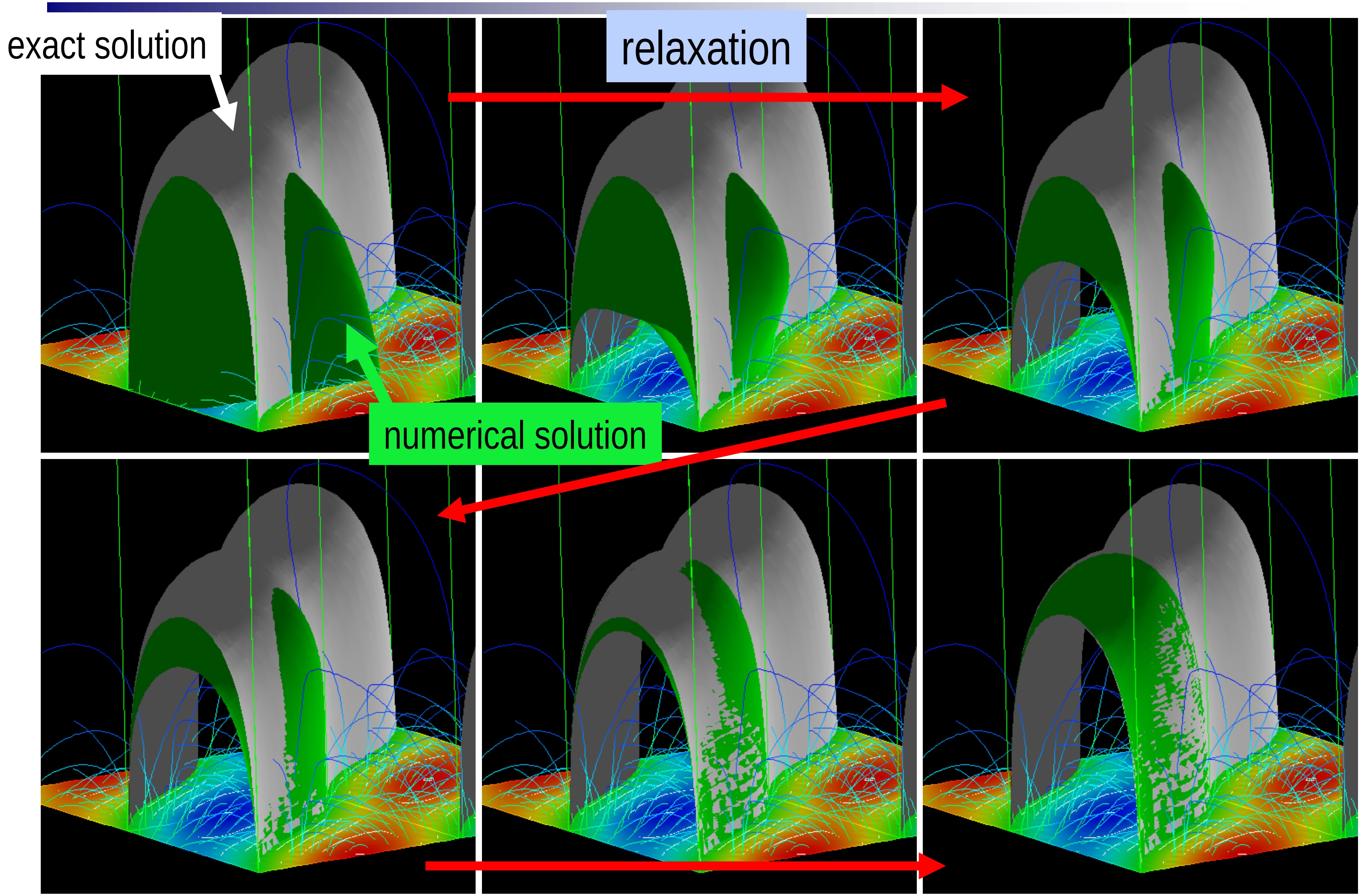
$$\eta \rightarrow 0 \ as \ \mathbf{V} \rightarrow 0$$

boundary conditions

$$B_x, B_y, B_z \,|_{\partial\Omega}$$



Relaxation method (test by Kusano)



Optimization method

- Wheatland-Sturrock-Roumeliotis 2000

$$L = \int_V [B^{-2} |\vec{j} \times \vec{B}|^2 + |\nabla \cdot \vec{B}|^2] dV$$

$$\left.\frac{\partial \vec{B}}{\partial t}\right|_{\mathrm{inter}} = \mu \vec{F}, (\mu > 0)$$

$$\left.\frac{\partial \vec{B}}{\partial t}\right|_{\mathrm{bound}} = 0$$

$$\frac{dL}{dt} = -2\int_V \frac{\partial \vec{B}}{\partial t} \cdot \vec{F} dV - 2\int_S \frac{\partial \vec{B}}{\partial t} \cdot \vec{G} dS$$

$$\frac{dL}{dt} = -2\int_V \mu F^2 dV$$

monotonically decrease

Wheatland et al. 2000

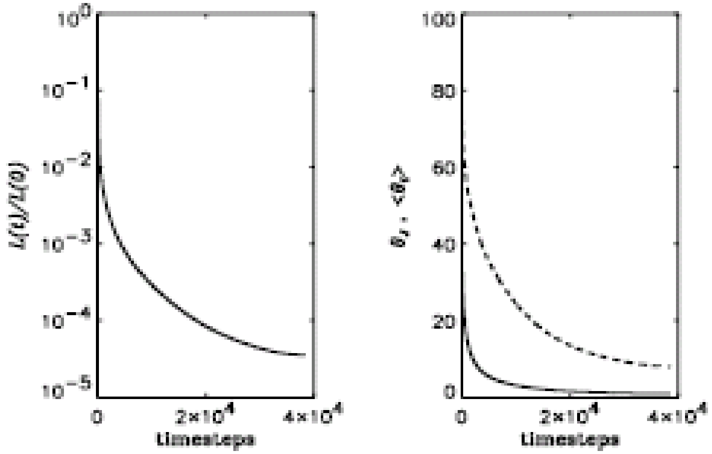


FIG. 1.—*Left*: Time history of the quantity *L* for the Low & Lou field calculation. *Right*: Time history of the current-weighted average angle between *B* and *J* (*solid line*) and of the average angle (*dashed line*).

Low and Lou's solution

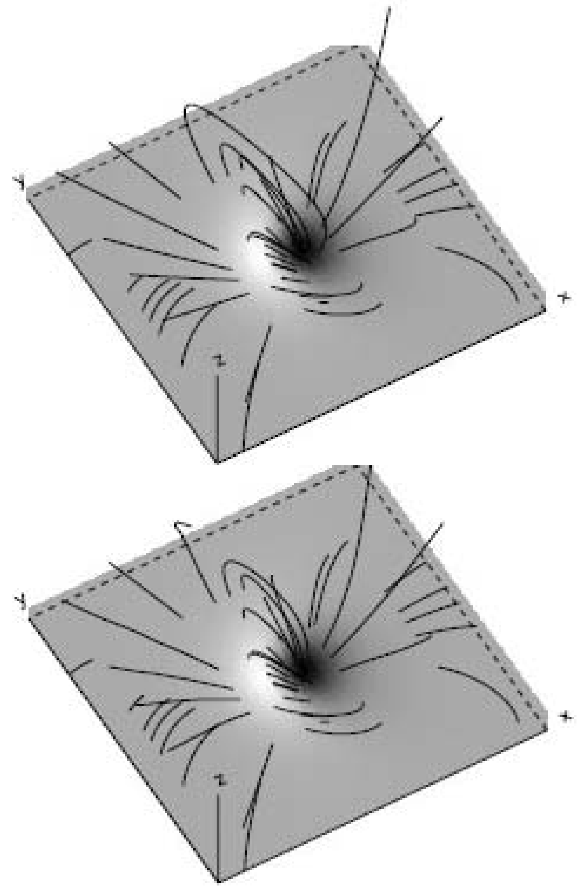
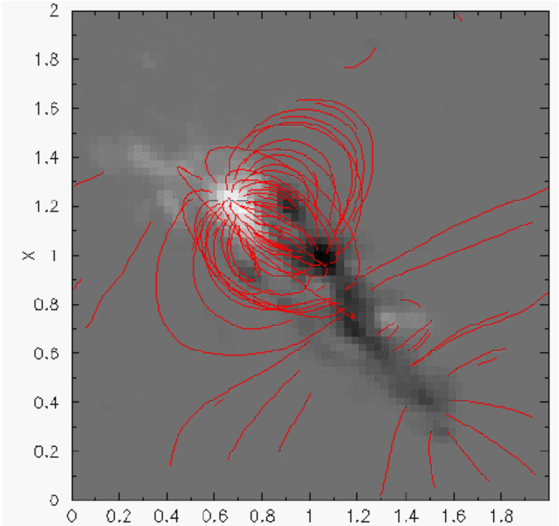
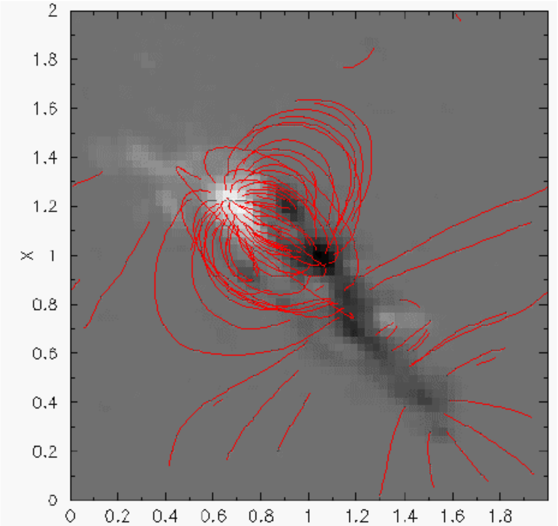


FIG. 3.—*Top*: Field lines calculated for the Low & Lou field. *Bottom*: Field lines for the reconstructed field.

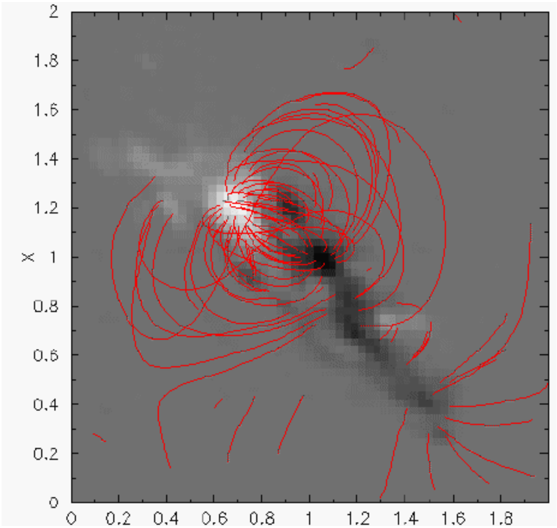
Nonlinear force-free (AR8100)



Potential

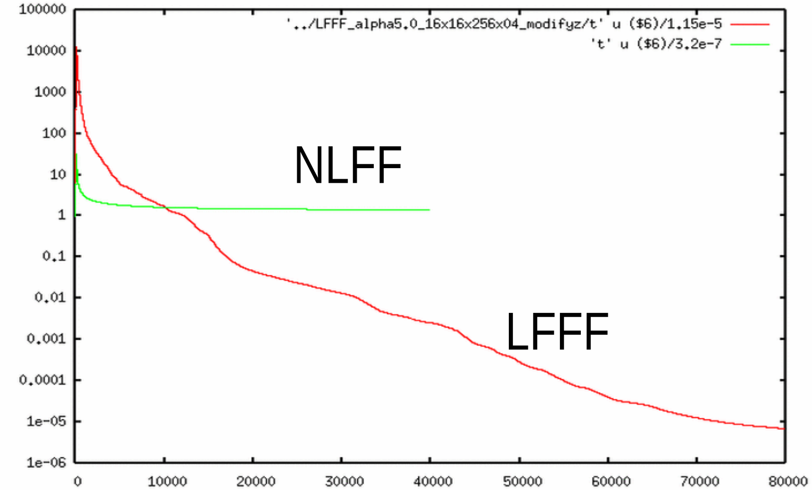


Linear force-free



Nonlinear force-free

<JxB>



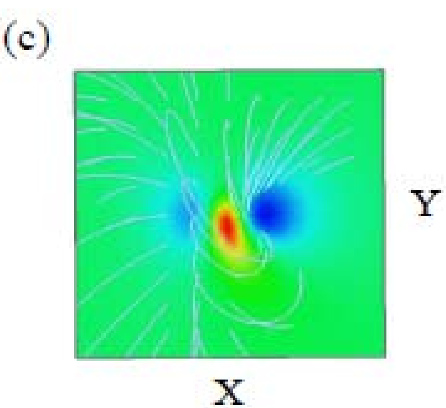
iteration loop

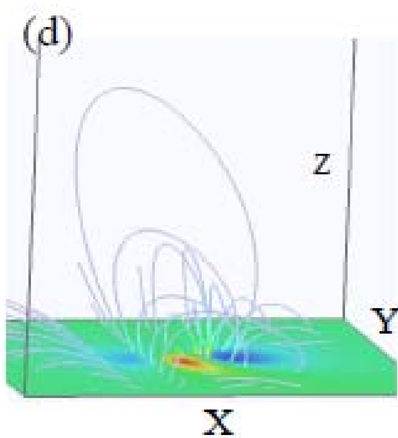
Problems

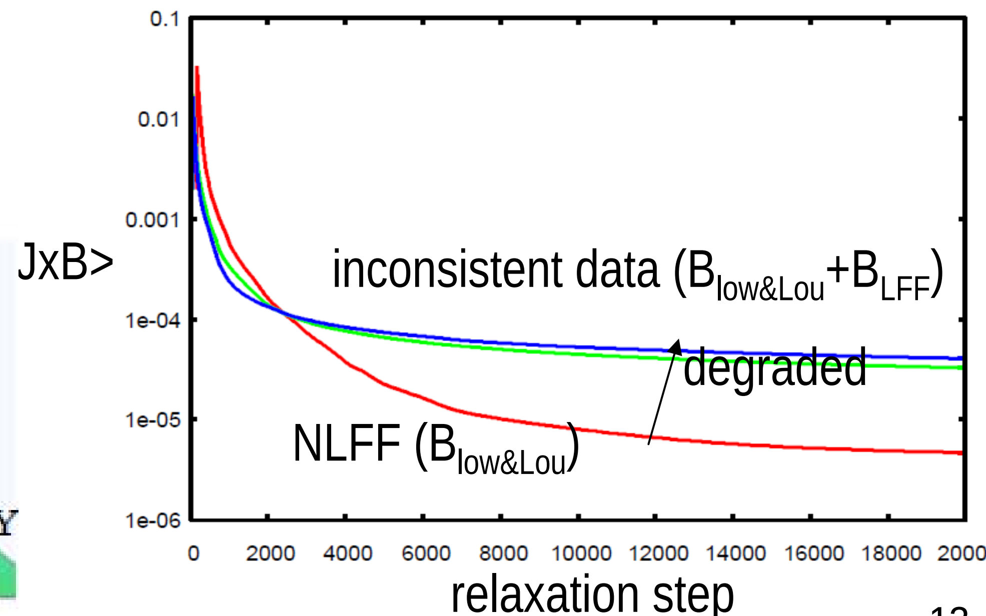
- inconsistency with force-free condition

method	consistency with F-F	data
Grad-Rubin	○	B_z & α (not B_{xy})
Relaxation-type	△	B_x B_y B_z

compromise between data and force-free condition







Wiegelmann, Inhester & Sakurai 2006

- Aly's virial theorem (1989)

total force balance

force-free condition ↔ data reliability

$$\int_S B_x B_z dxdy = \int_x B_y B_z dxdy = 0$$

$$\int_S (B_x^2 + B_y^2) dxdy = \int_S B_z^2 dxdy$$

total torque balance

$$\int_S x(B_x^2 + B_y^2) dxdy = \int_x x B_z^2 dxdy$$

$$\int_S y(B_x^2 + B_y^2) dxdy = \int_x y B_z^2 dxdy$$

$$\int_S y B_x B_z dxdy = \int_x x B_y B_z dxdy$$

preprocessing of magnetogram

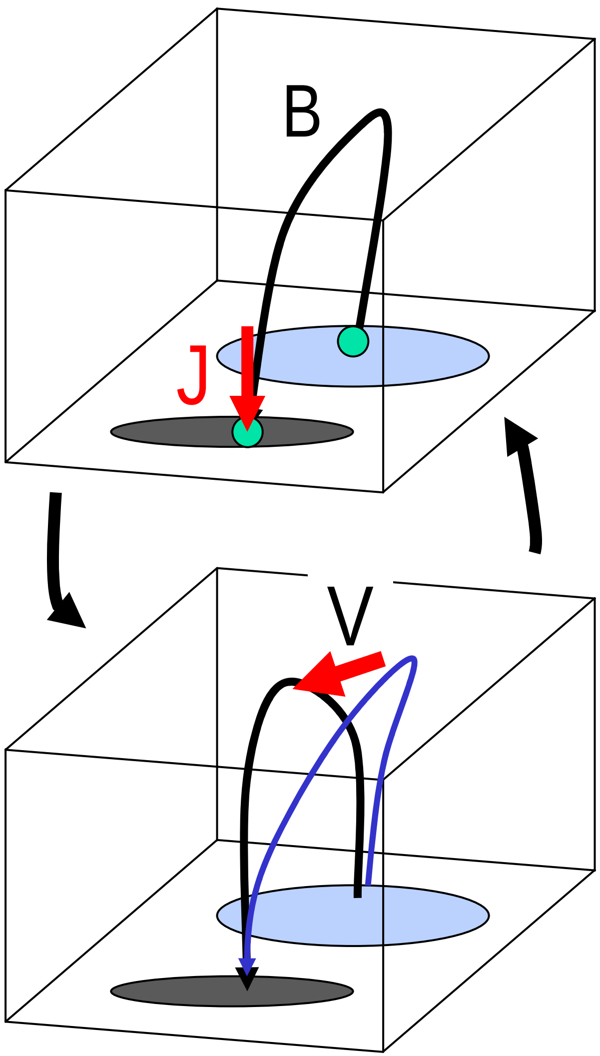
NLFF solver

Reconstruction of conjugate point

- field tracing from $\partial\Omega^{+} \rightarrow \partial\Omega^{-}$
  - $\rightarrow$ conjugate point
  - $\rightarrow$ mapping α
  - $\rightarrow$ $J_z = aB_z$
  - $\rightarrow$ Poisson eq. on $\partial\Omega^{-}$

$$\nabla^2 B_x = -\partial_x J_z - \partial_{yz} B_z$$

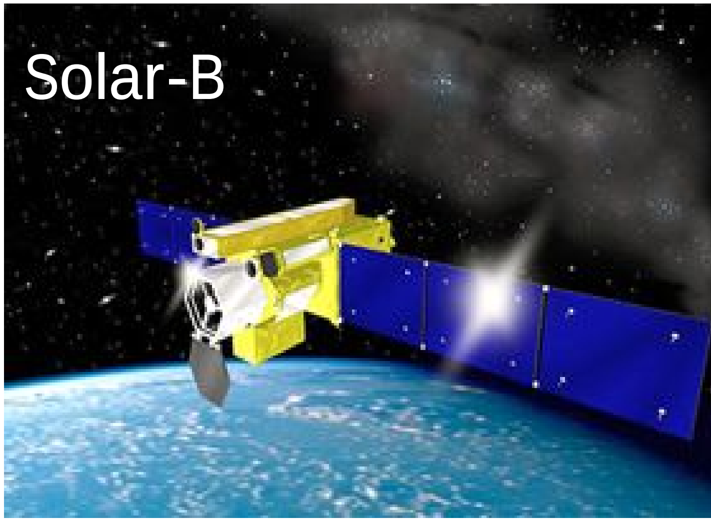
$$\nabla^2 B_y = -\partial_y J_z + \partial_{xz} B_z$$



- Relaxation of B

Summary

- Magnetic extrapolation is crucial to reveal the 3D structure and the stability of active region magnetic field.
- Several methods have been recently developed, and they can provide the solution as long as the data are consistent with the force-free equation.
- Issues
  - compromise between data and force-free condition
  - non-force free effects (pressure, gravity, flow)
    → full-*MHD modeling*
  - condition on the other boundaries
    → *combination with global model*
  - convergence speed
    → *parallel computation*



40 TFlops
10 TB
(5120 cpu)

- data preprocessing
- NLFF extrapolation
- multi-resolution modeling