Hanle effect on Lyman-α line

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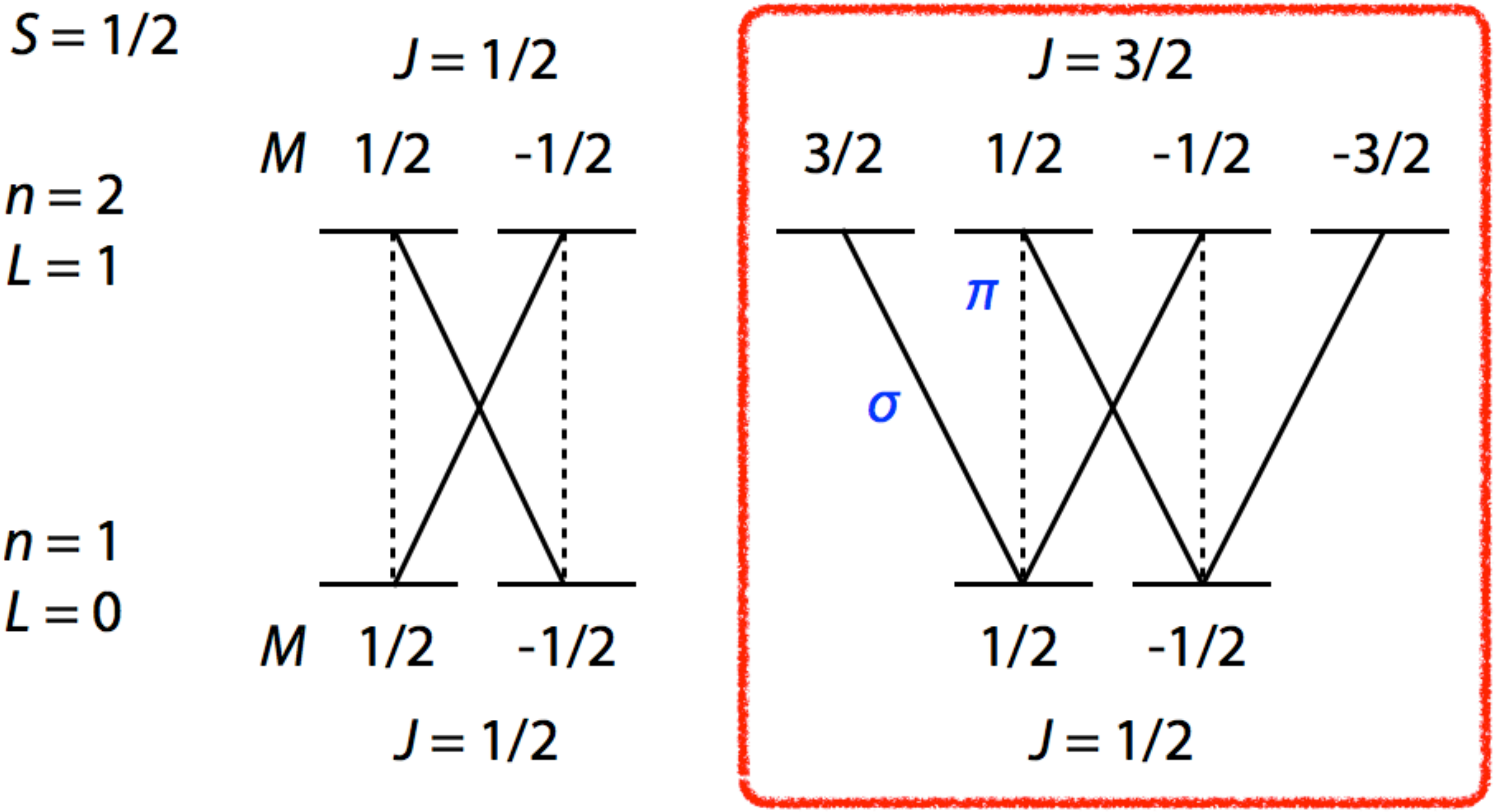
states concerning Lyman α



density matrix theory

- eigenstates of J_z, $|M\rangle$, are considered and density matrix (operator) is expressed as

$$\rho = \sum_M p_M |M\rangle\langle M|$$

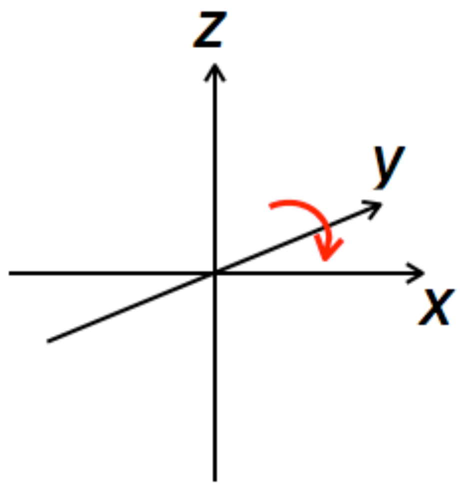
- isotropic case

$$\rho = \frac{1}{4}\left\{\left|\frac{3}{2}\right\rangle\left\langle\frac{3}{2}\right| + \left|\frac{1}{2}\right\rangle\left\langle\frac{1}{2}\right| + \left|-\frac{1}{2}\right\rangle\left\langle-\frac{1}{2}\right| + \left|-\frac{3}{2}\right\rangle\left\langle-\frac{3}{2}\right|\right\}$$

$$= \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- off diagonal components appear when aniso-tropic (polarized) and there exists coherence among basis states

- change of quantization axis is performed as rotation of coordinates



$$\rho_{MN} = \langle M|\rho|N\rangle \quad \text{(}z\text{-axis domain)}$$

$$\rho_{M_xN_x} = \langle M_x|\rho|N_x\rangle \quad \text{(}x\text{-axis domain)}$$

$$= (\langle M|\mathscr{D}^\dagger(R))\rho(\mathscr{D}(R)|N\rangle)$$

$$= \sum_{mn}\langle M|\mathscr{D}^\dagger(R)|m\rangle\langle m|\rho|n\rangle\langle n|\mathscr{D}(R)|N\rangle$$

$$= \sum_{mn}\mathscr{D}^{(J)*}_{Mm}(R)\mathscr{D}^{(J)}_{nN}(R)\langle m|\rho|n\rangle$$

Euler rotation

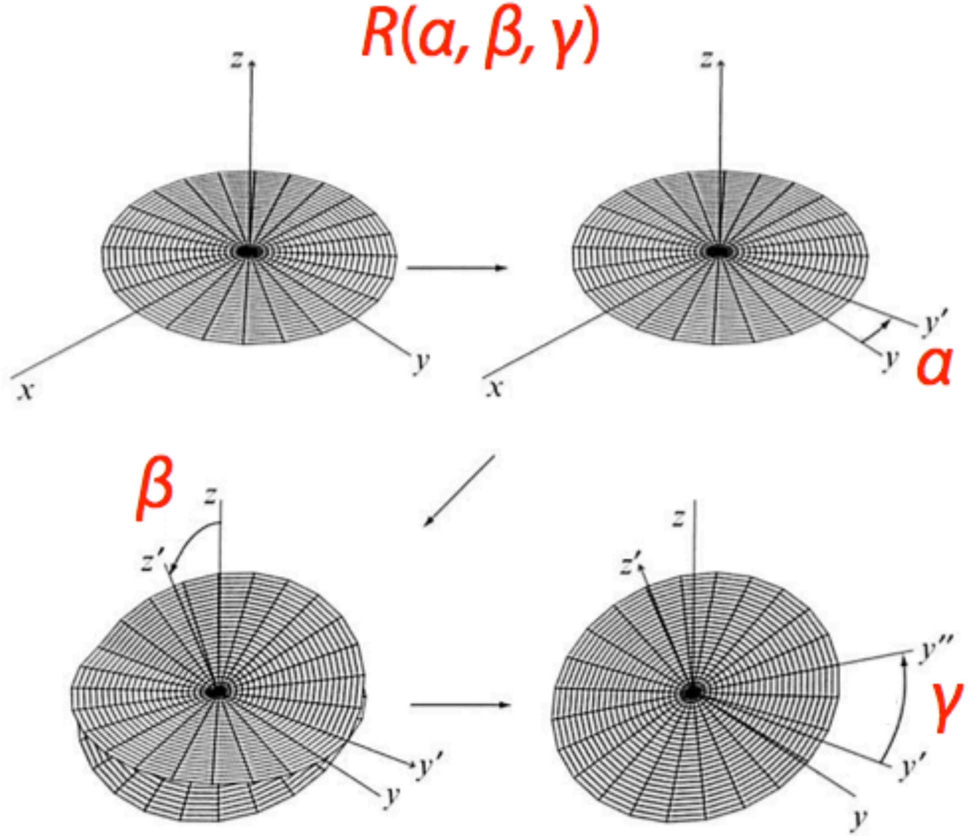


Fig. B.1. Three steps of Euler rotations. Fist, we rotate the rigid body about the z-axis by angle ϕ. The second rotation is performed about the y'-axis, which is the body-fixed y-axis after the first rotation, by angle θ. The third rotation is about the z'-axis by angle γ. The body y-axis now becomes the y''-axis [2]

Fujimoto, *Plasma polarization spectroscopy*, Springer

rotation operator (matrix)

$$\langle M|\mathscr{D}(\alpha,\beta,\gamma)|N\rangle = \mathscr{D}^{(J)}_{MN}(\alpha,\beta,\gamma) = e^{-i(M\alpha+N\gamma)}d^{(J)}_{MN}(\beta)$$

$$d^{(J)}_{MN}(\beta) = \sum_k (-1)^{k-M+N}\frac{\sqrt{(J+M)!(J-M)!(J+N)!(J-N)!}}{(J+M-k)!k!(J-k-N)!(k-M+N)!}$$
$$\times\left(\cos\frac{\beta}{2}\right)^{2J-2k+M-N}\left(\sin\frac{\beta}{2}\right)^{2k-M+N}$$

Wigner's formula

- for pure state of $|3/2\rangle$

$$\rho_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- when x-axis is taken as quantization axis

$$\rho_x = \frac{1}{8} \begin{pmatrix} 1 & -\sqrt{3} & \sqrt{3} & -1 \\ -\sqrt{3} & 3 & -3 & \sqrt{3} \\ \sqrt{3} & -3 & 3 & -\sqrt{3} \\ -1 & \sqrt{3} & -\sqrt{3} & 1 \end{pmatrix}$$

- isotropic matrix is unchanged by rotation

equation of motion

- density matrix is derived as solution of equation of motion
$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t}\rho = [H_\mathrm{F}, \rho]$$
- Hamiltonian H_F consists of perturbation due to magnetic field

$$\begin{aligned}\langle M|H_\mathrm{F}|N\rangle &= -\mu_\mathrm{B} g_J B \langle M|J_z|N\rangle \\ &= -\mu_\mathrm{B} g_J B M \delta_{MN} \\ &= -\hbar\omega_0 M \delta_{MN}\end{aligned}$$

- μ_B and g_J are Bohr magneton and Landé g-factor, respectively, and $\omega_0 M$ corresponds to Larmor angular frequency

line intensity

- linear polarization components are derived and polarization degree is evaluated

$$I_{\mathbf{e}} = C_{\mathrm{D}} \mathrm{Tr}[\mathfrak{D}(\beta G; \mathbf{e})\rho]$$

$$= C_{\mathrm{D}} \sum_{MM'} \langle \alpha JM | \mathbf{e} \cdot \mathbf{d} \sum_{N} |\beta GN\rangle \langle \beta GN | \mathbf{e}^* \cdot \mathbf{d} | \alpha JM' \rangle \langle \alpha JM' | \rho | \alpha JM \rangle$$

$$\mathbf{e} \cdot \mathbf{d} = \mathbf{e}^* \cdot \mathbf{d} = \begin{cases} d_0 & \text{(in } z \text{ axis)} \\ -\frac{1}{\sqrt{2}}(d_1 - d_{-1}) & \text{(in } x \text{ axis)} \\ \frac{i}{\sqrt{2}}(d_1 + d_{-1}) & \text{(in } y \text{ axis)} \end{cases}$$

$$\langle \alpha JM | d_q | \beta GN \rangle = (-1)^{\frac{3}{2} - M} \begin{pmatrix} \frac{3}{2} & 1 & \frac{1}{2} \\ -M & q & N \end{pmatrix} \langle \alpha J || d || \beta G \rangle$$

Wigner-Eckart theorem

conditions for calculation

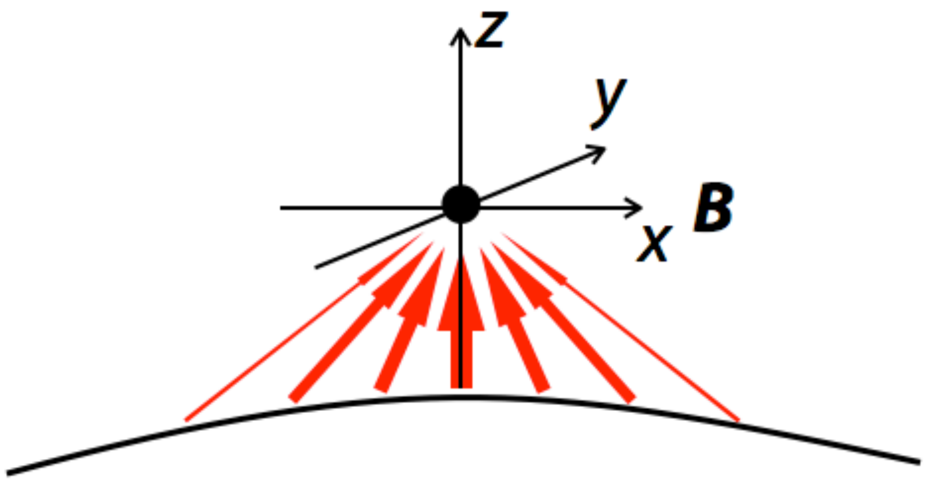
- excitation takes place only through photo-absorption
- $J = 3/2$ states are only considered
- hyperfine structure and wavefunction mixing with $J = 1/2$ due to Zeeman effect are not taken into consideration
- B field is assumed to be in the x-axis (ζ-axis, later)
- instantaneous excitation and subsequent radiative decay are considered
- collisional relaxation is ignored

strategy

- initial density matrix is determined by photo-absorption probability which is integrated over photosphere
- quantization axis is changed to *B* direction (ζ-axis) and density matrix is transferred accordingly
- equation of motion is solved with initial condition above
- linear polarization components in ζ- and ξ-axis are evaluated with the density matrix obtained
- *B*-dependence of polarization degree is evaluated from time-integrated radiation intensity

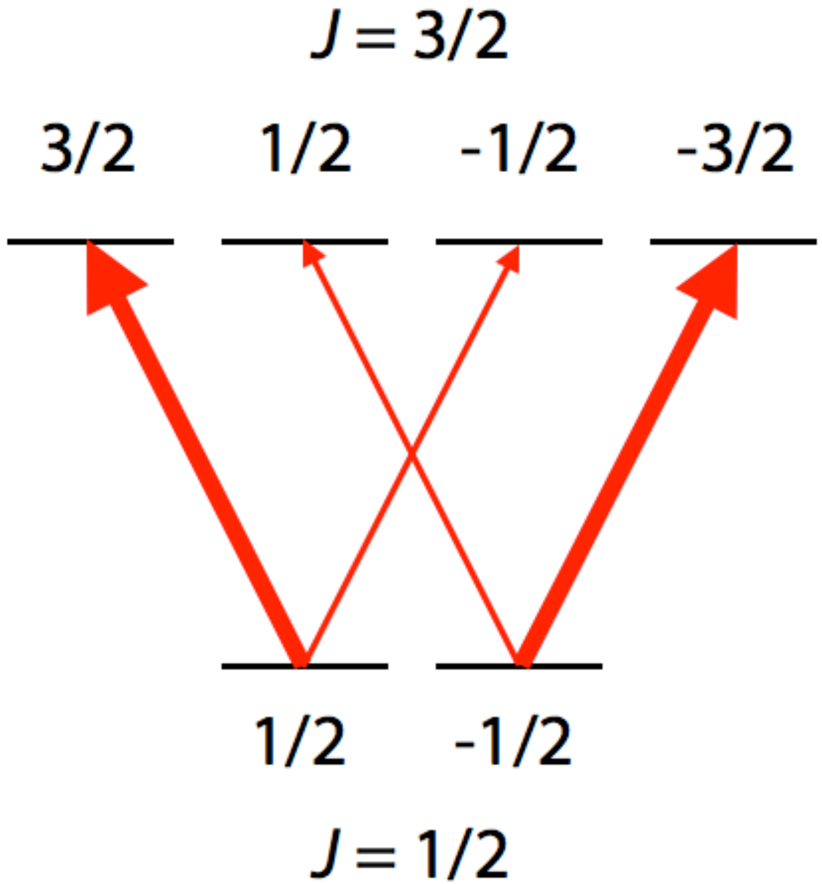
photo-excitation

- quantization axis (z-axis) is taken to be normal to the solar surface
- superposition of unidirectional irradiation is considered
- each irradiation component induces incoherent σ-light excitation if quantization axis taken in light propagation direction



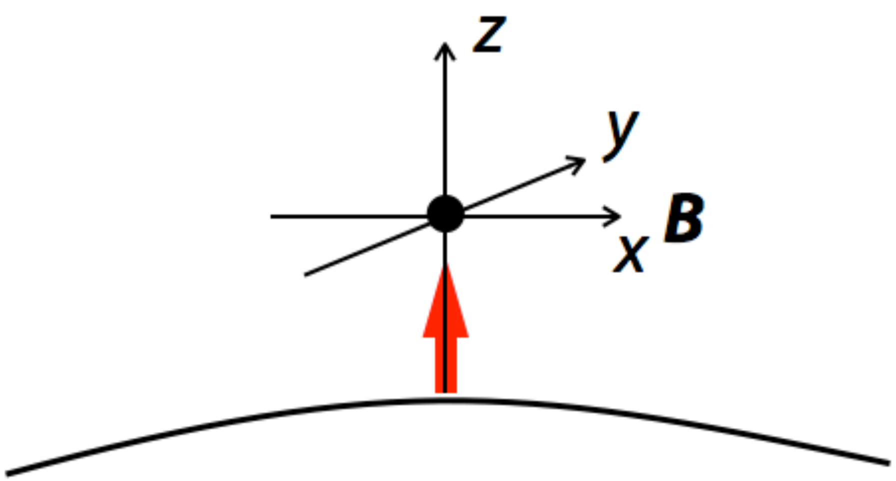
σ-light excitation

- incoherent σ-light excitation produces incoherent mixed states



irradiation in z-axis direction

$$\rho(t=0) = \frac{1}{8}\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

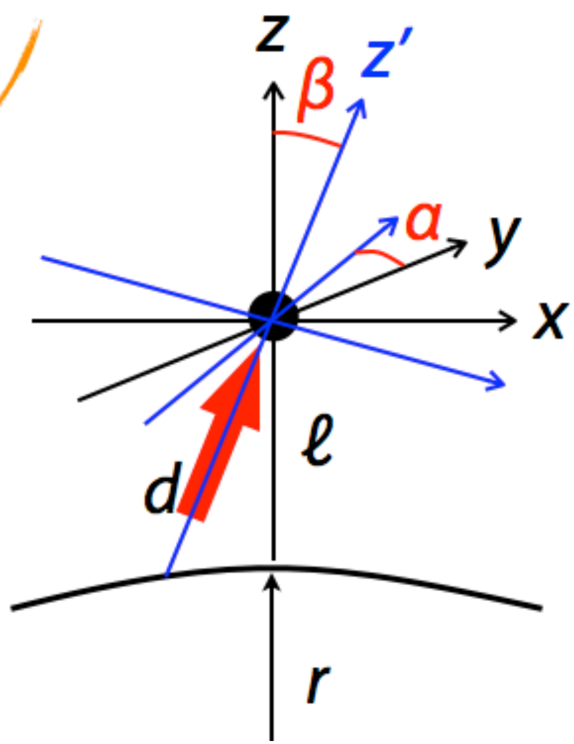


- each matrix element is obtained by integration over the solar surface

$$\langle M|\rho_z|M'\rangle = \int_0^{\beta_{max}} \int_0^{2\pi} \frac{r\sin\beta}{2d} \sqrt{\frac{r^2-(r+\ell)^2\sin^2\beta}{r^2-d^2\sin^2\beta}} \langle M|\rho_{z'}|M'\rangle d\alpha d\beta$$

$$\langle M|\rho_{z'}|M'\rangle = \sum_{mm'} (-1)^{M'-m'} d^{(\frac{3}{2})}_{Mm}(\beta) d^{(\frac{3}{2})}_{-M'-m'}(\beta) \langle m|_{z'}\rho_{z'}|m'\rangle_{z'} \delta_{MM'}$$

$$\rho_{z'} = \frac{1}{8}\begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

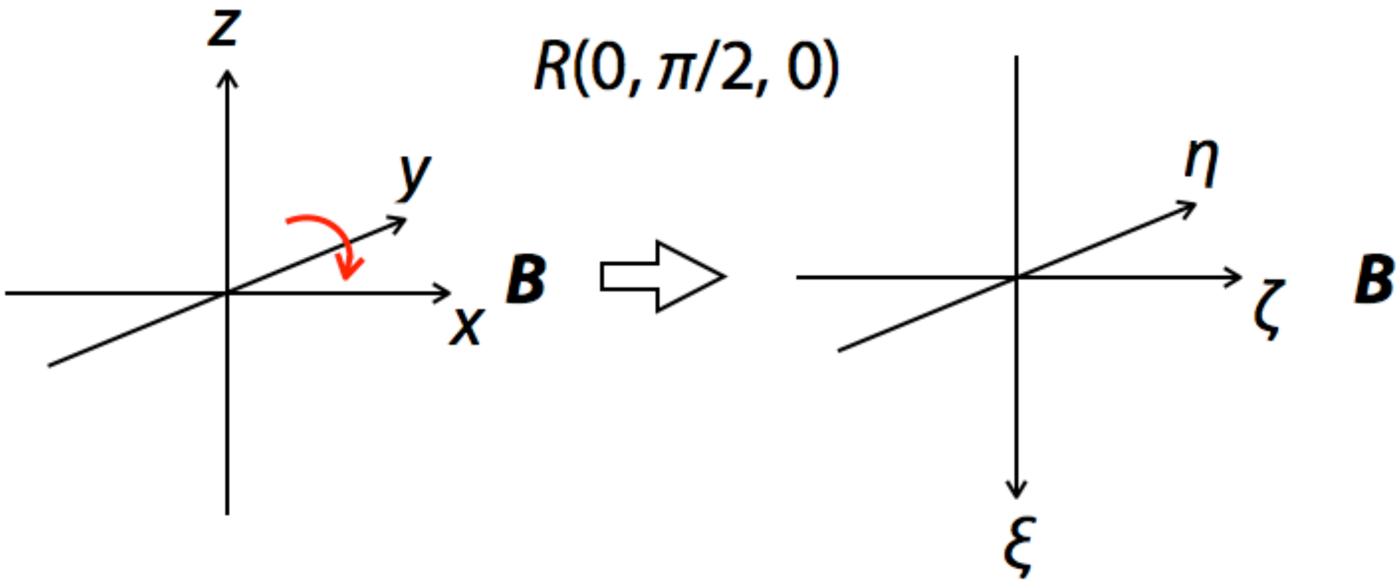


$$d = (r+\ell)\cos\beta - \sqrt{r^2-(r+\ell)^2\sin^2\beta}$$

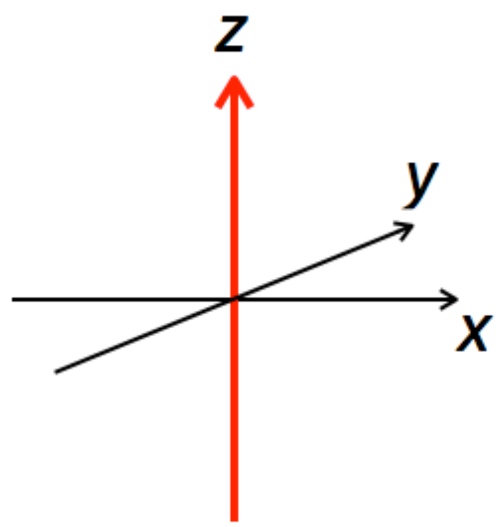
$$\beta_{max} = \frac{\pi}{2} - \cos^{-1}\left(1-\frac{\ell}{r}\right)$$

initial density matrix

- integral is calculated with $r = 6.96 \times 10^8$ m and $\ell \sim 3 \times 10^6$ m
- ρ is found to be diagonal after integration
- quantization axis is changed to x-axis or ζ-axis (B direction)

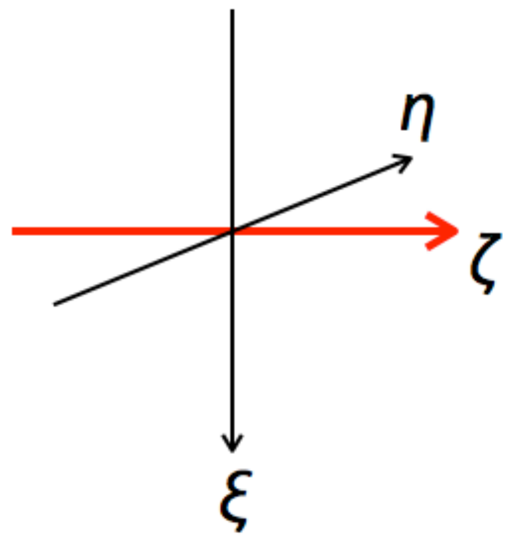


$$\rho_z(t=0) = \begin{pmatrix} \rho_1 & 0 & 0 & 0 \\ 0 & \rho_2 & 0 & 0 \\ 0 & 0 & \rho_2 & 0 \\ 0 & 0 & 0 & \rho_1 \end{pmatrix}$$



$$\Downarrow$$

$$\rho_\zeta(t=0) = \begin{pmatrix} c_1 & 0 & c_3 & 0 \\ 0 & c_2 & 0 & c_3 \\ c_3 & 0 & c_2 & 0 \\ 0 & c_3 & 0 & c_1 \end{pmatrix}$$



equation of motion

- equation of motion is solved with initial condition derived above

$$i\hbar \frac{\mathrm{d}}{\mathrm{d}t}\rho_\zeta = \left[H_\mathrm{F}, \rho_\zeta\right]$$

$$\begin{aligned}\langle M|H_\mathrm{F}|N\rangle &= -\mu_\mathrm{B} g_J B \langle M|J_z|N\rangle \\ &= -\mu_\mathrm{B} g_J B M \delta_{MN} \\ &= -\hbar\omega_0 M\end{aligned}$$

$$H_\mathrm{F} = -\hbar\omega_0 \begin{pmatrix} \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}$$

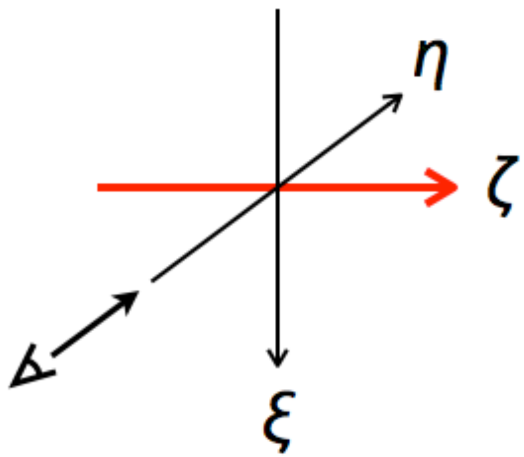
- equation is explicitly written for general ρ

$$
\begin{aligned}
i\hbar \frac{\mathrm{d}}{\mathrm{d}t}\rho_\zeta &= [H_\mathrm{F}, \rho_\zeta] \\
&= -\hbar\omega_0 \begin{pmatrix} 0 & \rho_{31} & 2\rho_{3-1} & 3\rho_{3-3} \\ -\rho_{13} & 0 & \rho_{1-1} & 2\rho_{1-3} \\ -2\rho_{-13} & -\rho_{-11} & 0 & \rho_{-1-3} \\ -3\rho_{-33} & -2\rho_{-31} & -\rho_{-3-1} & 0 \end{pmatrix}
\end{aligned}
$$

- $\rho_\zeta(t)$ is readily obtained with initial condition

$$
\rho(t) = \begin{pmatrix} c_1 & 0 & c_3 e^{2i\omega_0 t} & 0 \\ 0 & c_2 & 0 & c_3 e^{2i\omega_0 t} \\ c_3 e^{-2i\omega_0 t} & 0 & c_2 & 0 \\ 0 & c_3 e^{-2i\omega_0 t} & 0 & c_1 \end{pmatrix}
$$

line intensity



- linear polarization components in ζ-axis and ξ-axis are respectively determined

$$
\begin{aligned}
I_{\mathbf{e}} &= C_{\mathrm{D}} \mathrm{Tr}[\mathfrak{D}(\beta G; \mathbf{e})\rho] \\
&= C_{\mathrm{D}} \sum_{MM'} \langle \alpha JM | \mathbf{e} \cdot \mathbf{d} \sum_{N} | \beta GN \rangle \langle \beta GN | \mathbf{e}^* \cdot \mathbf{d} | \alpha JM' \rangle \langle \alpha JM' | \rho | \alpha JM \rangle
\end{aligned}
$$

$$
I_\zeta(t) = C_{\mathrm{D}} \frac{c_2}{3} \left| \langle \alpha J || d || \beta G \rangle \right|^2 e^{-g_0 t}
$$

$$
I_\xi(t) = C_{\mathrm{D}} \left(\frac{3c_1 + c_2}{12} - \frac{c_3}{2\sqrt{3}} \cos 2\omega_0 t \right) \left| \langle \alpha J || d || \beta G \rangle \right|^2 e^{-g_0 t}
$$

here, spontaneous decay factor $e^{-g_0 t}$ is added

- finally, time integrated line intensity is derived

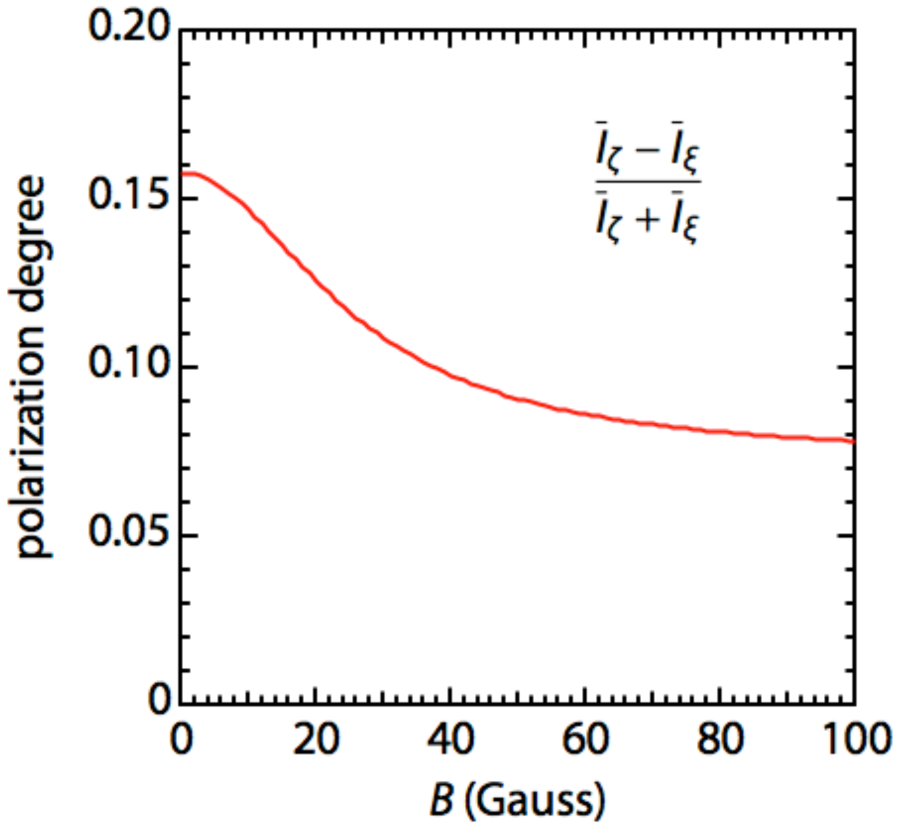
$$\bar{I}_\zeta = \int_0^\infty I_\zeta(t)\mathrm{d}t = C_\mathrm{D}\frac{c_2}{3g_0}\,|\langle aJ||d||\beta G\rangle|^2$$

$$\begin{aligned}\bar{I}_\xi &= \int_0^\infty I_\xi(t)\mathrm{d}t \\ &= C_\mathrm{D}\frac{1}{g_0}\left[\frac{3c_1+c_2}{12} - \frac{c_3}{2\sqrt{3}}\frac{1}{1+(2\omega_0/g_0)^2}\right]|\langle aJ||d||\beta G\rangle|^2\end{aligned}$$

- polarization degree P is evaluated as

$$P = \frac{\bar{I}_\zeta - \bar{I}_\xi}{\bar{I}_\zeta + \bar{I}_\xi}$$

polarization degree



problems to be solved

- influence of hyperfine structure and wavefunction mixing with $J = 1/2$ due to Zeeman effect
- relaxation due to collisional transitions
- polarization degree over line profile — radiation transport?
- influence of line-integral effect

THE HANLE EFFECT
OF THE CORONAL Lα LINE OF HYDROGEN:
THEORETICAL INVESTIGATION

Solar Physics **78** (1982) 157–178.

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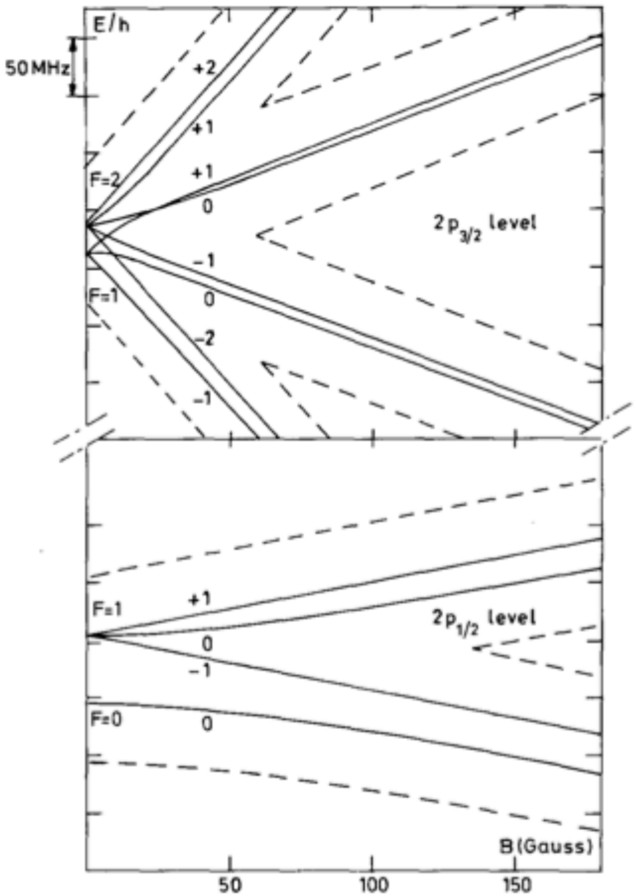

Fig. 1. Effect of the magnetic field on the hyperfine structure levels of the upper levels $2p_{1/2,3/2}$ of the Lα line. In abscissa is the field strength in gauss, in ordinate is the energy in MHz. On each sublevel the value of the magnetic quantum number m_F is reported. The sublevels are broadened by their natural width (dotted lines). It can be seen that the hyperfine splitting and the natural width are of the same order of magnitude.

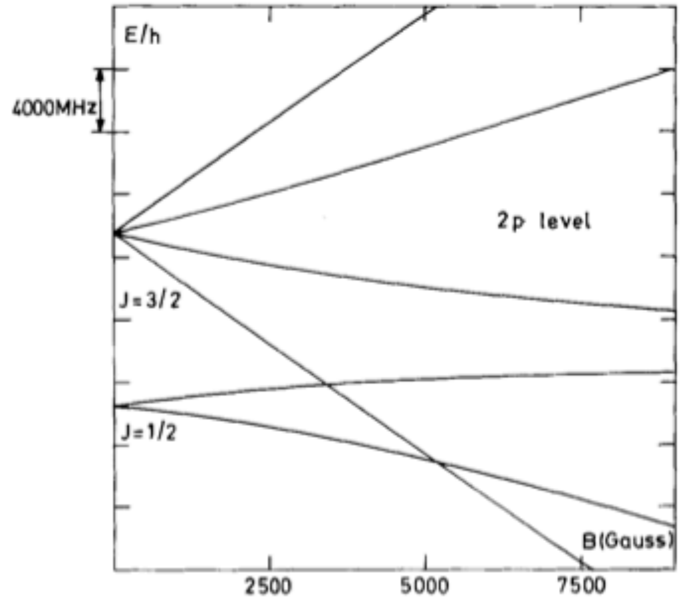

Fig. 2. The same as Figure 1, but for stronger magnetic field strengths. The hyperfine structure and the natural width are not resolved. It can be seen that beyond 600 gauss, which is the domain of sensitivity to the Hanle effect, the fine structure levels are well separated and the coupling coherences are negligible.