球面テンソルと球面調和関数の類似性と、それによる非等方放射場の分類

2012/05/28 R. Kano (NAOJ)

What is $\overline{J_0}^2$?

$$\mathcal{A} = \frac{\bar{J}_0^2}{\bar{J}_0^0},$$

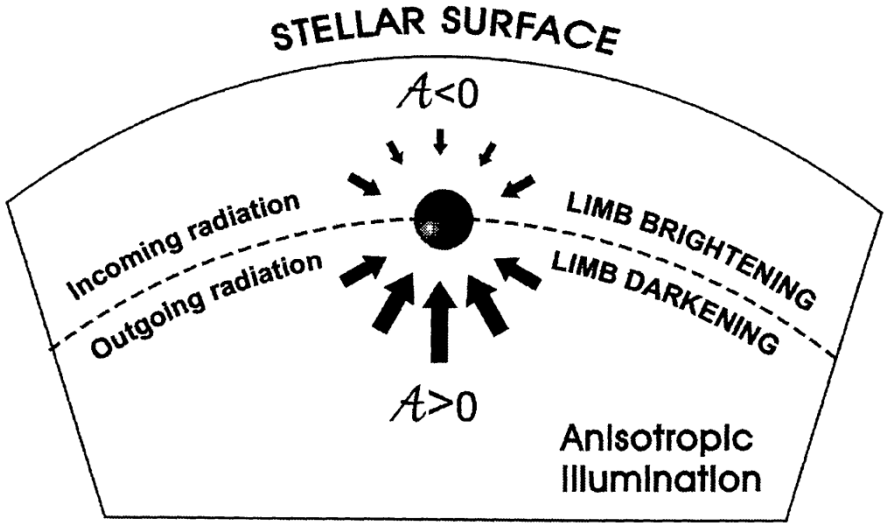


Figure 5. The anisotropic illumination in a stellar atmosphere.

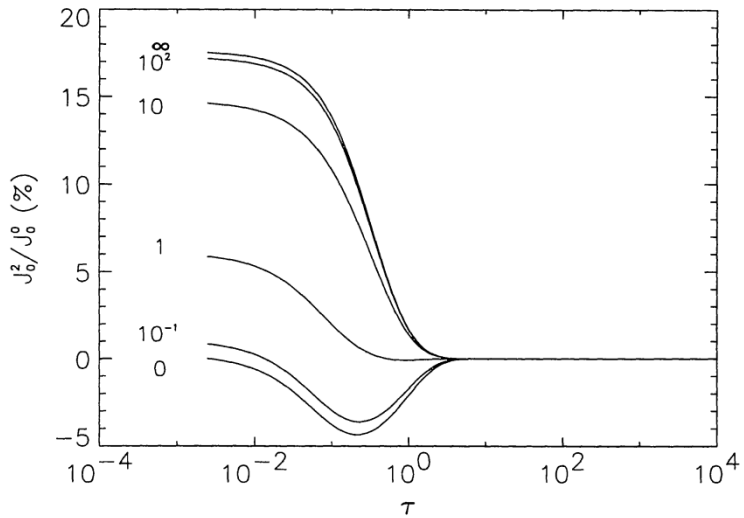
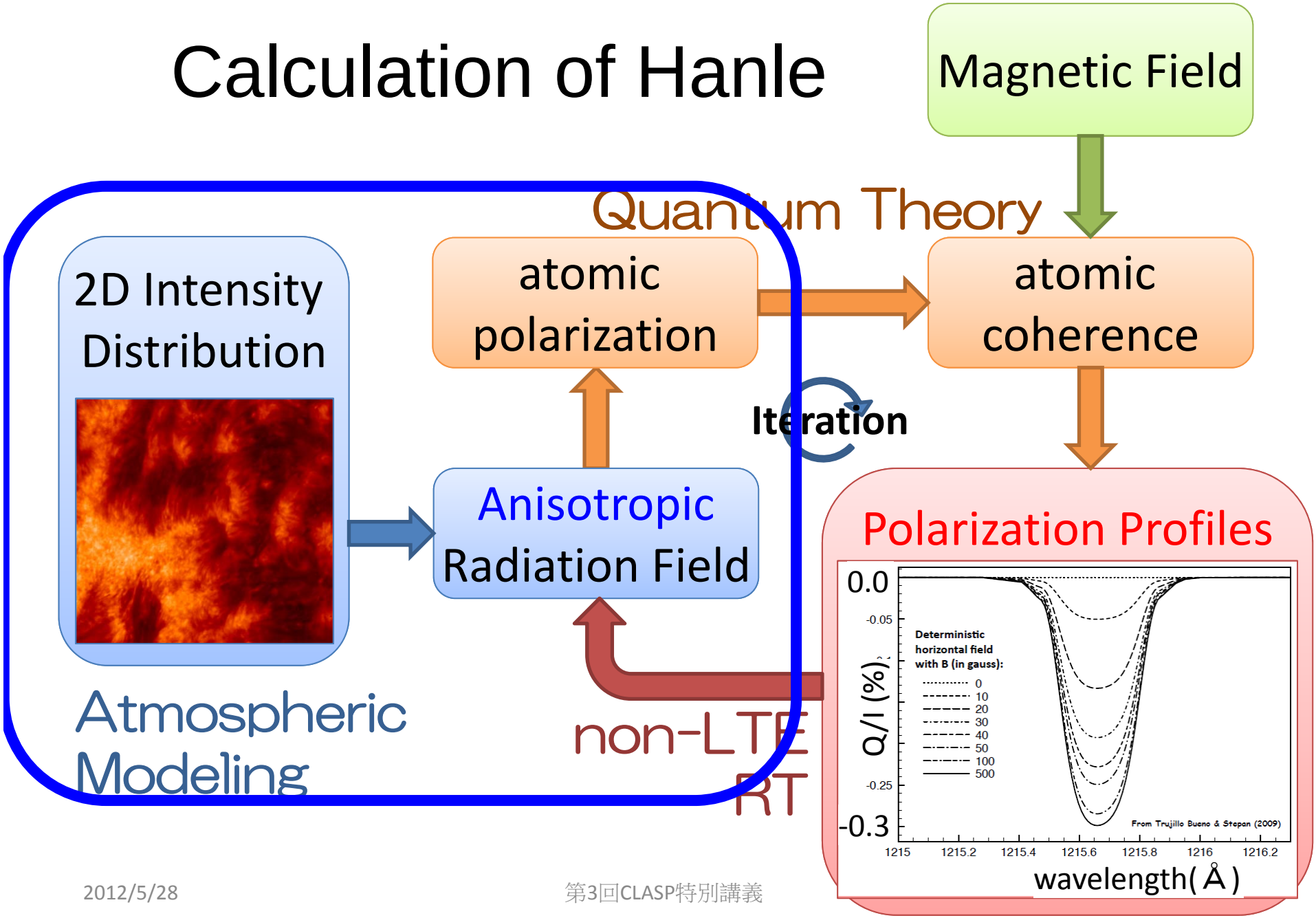


Figure 4. The anisotropy factor in Milne–Eddington atmospheres.

Calculation of Hanle



Stokes・密度関数・放射場の関係

2012/1/19 ハンレ特別講義・資料より

Stokes parameters

$$\tilde{\varepsilon}_i(\vec{\Omega}) = \frac{h^2\nu^4}{2\pi c^2}\,\mathcal{N}\,(2J_u+1)\,B(\alpha_u J_u \to \alpha_\ell J_\ell)$$
$$\times \sum_{KQ} \sqrt{3}\,(-1)^{1+J_\ell+J_u} \begin{Bmatrix} 1 & 1 & K \\ J_u & J_u & J_\ell \end{Bmatrix} \mathcal{T}^K_Q(i,\vec{\Omega})\,\rho^K_Q(\alpha_u J_u)$$

密度関数

$$\rho^K_Q(\alpha_u J_u) = \sqrt{\frac{2J_\ell+1}{2J_u+1}}\,\frac{B(\alpha_\ell J_\ell \to \alpha_u J_u)}{A(\alpha_u J_u \to \alpha_\ell J_\ell) + 2\pi\mathrm{i}\,\nu_\mathrm{L}\,g_{\alpha_u J_u}\,Q}$$
$$\times\, w^{(K)}_{J_u J_\ell}\,(-1)^Q\,J^K_{-Q}(\nu_0)\;\rho^0_0(\alpha_\ell J_\ell)$$

Stokes parameters

Isotropic radiation

$$J^K_Q(\nu) = \oint \frac{\mathrm{d}\Omega}{4\pi}\,\mathcal{I}^K_Q(\nu,\vec{\Omega}) = \oint \frac{\mathrm{d}\Omega}{4\pi} \sum_{i=0}^{3} \mathcal{T}^K_Q(i,\vec{\Omega})\,S_i(\nu,\vec{\Omega})$$

geometrical factors

Spherical tensors and Spherical harmonics

Spherical tensors

$$\bar{J}^0_0 = \int \mathrm{d}x \oint \phi_x \frac{\mathrm{d}\vec{\Omega}}{4\pi} I_{x\vec{\Omega}},$$

$$\bar{J}^1_0 = \int \mathrm{d}x \oint \phi_x \frac{\mathrm{d}\vec{\Omega}}{4\pi} \sqrt{\frac{3}{2}}\mu V_{x\vec{\Omega}},$$

$$\bar{J}^1_1 = -\int \mathrm{d}x \oint \phi_x \frac{\mathrm{d}\vec{\Omega}}{4\pi} e^{i\chi} \sqrt{\frac{3}{2}}\sqrt{1-\mu^2} V_{x\vec{\Omega}},$$

$$\boxed{\bar{J}^2_0 = \int \mathrm{d}x \oint \phi_x \frac{\mathrm{d}\vec{\Omega}}{4\pi} \frac{1}{2\sqrt{2}} \left[(3\mu^2-1) I_{x\vec{\Omega}} + 3(\mu^2-1) Q_{x\vec{\Omega}}\right],}$$

$$\bar{J}^2_1 = \int \mathrm{d}x \oint \phi_x \frac{\mathrm{d}\vec{\Omega}}{4\pi} \frac{\sqrt{3}}{2} e^{i\chi} \sqrt{1-\mu^2} \left[-\mu\left(I_{x\vec{\Omega}} + Q_{x\vec{\Omega}}\right) - iU_{x\vec{\Omega}}\right],$$

$$\bar{J}^2_2 = \int \mathrm{d}x \oint \phi_x \frac{\mathrm{d}\vec{\Omega}}{4\pi} \frac{\sqrt{3}}{2} e^{2i\chi} \left[\frac{1}{2}(1-\mu^2) I_{x\vec{\Omega}} - \frac{1}{2}(1+\mu^2) Q_{x\vec{\Omega}} - i\mu U_{x\vec{\Omega}}\right]$$

μ=cosθ

Spherical harmonics

$$Y^0_0 = \frac{1}{\sqrt{4\pi}}$$

$$Y^0_1 = \sqrt{\frac{3}{4\pi}} \cos\theta$$

$$Y^{\pm 1}_1 = \mp\sqrt{\frac{3}{8\pi}} \sin\theta\, e^{\pm i\phi}$$

$$Y^0_2 = \sqrt{\frac{5}{16\pi}} (3\cos^2\theta - 1)$$

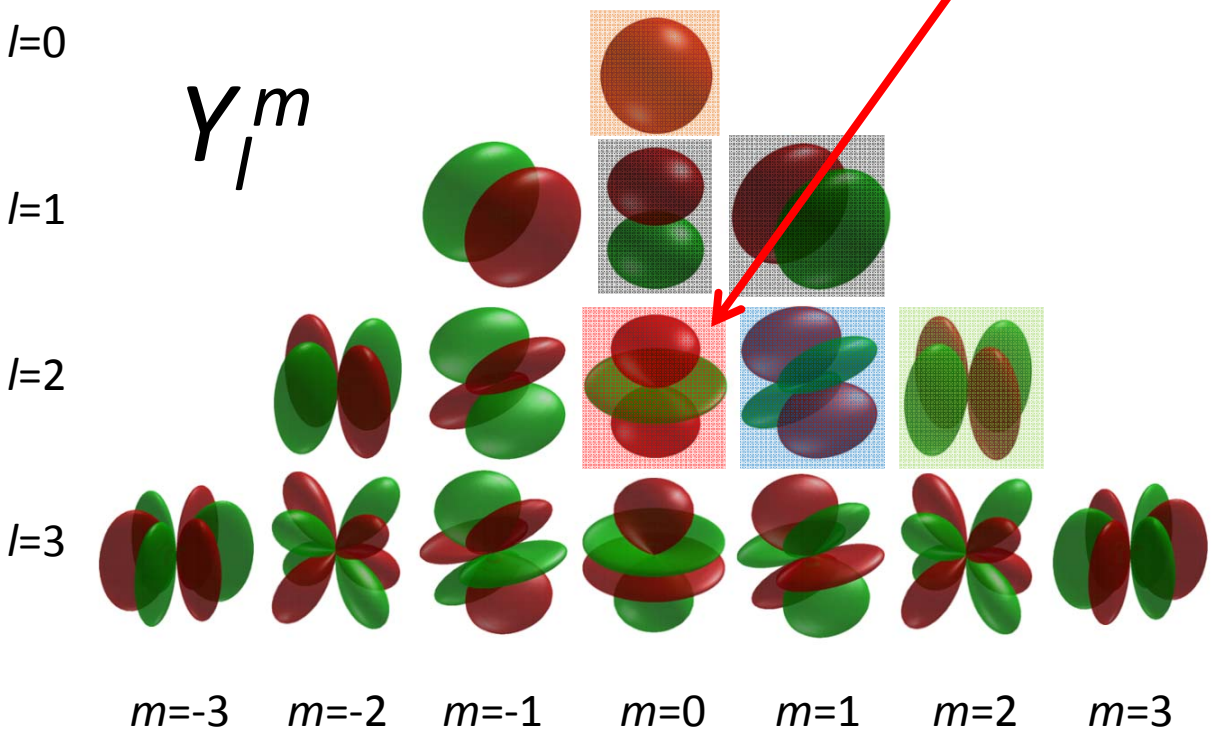
$$Y^{\pm 1}_2 = \mp\sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta\, e^{\pm i\phi}$$

$$Y^{\pm 2}_2 = \sqrt{\frac{15}{32\pi}} \sin^2\theta\, e^{\pm 2i\phi}$$

Shape of Spherical harmonics

$\bar{J}_0^2$

Y_l^m



$$Y_0^0 = \frac{1}{\sqrt{4\pi}}$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{\pm i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$$

$$Y_2^{\pm 1} = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{\pm i\phi}$$

$$Y_2^{\pm 2} = \sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{\pm 2i\phi}$$

Anisotropy of the radiation in the calculation.

Emission and absorption coefficients

$$\varepsilon_i(\nu, \mathbf{\Omega}) = \sum_{nmr} \sum_{qq'} (-1)^{q+q'} \frac{16\pi^3 e_0^2}{c^3} \nu_{nr} \nu_{mr} \nu^2 \mathcal{N} \times$$

$$\times \mathrm{Re}\{(r_q)_{nr}(r_{q'})^*_{mr} \rho_{mn} \Phi(\nu_{nr} - \nu) \mathcal{T}_{-q-q'}(i, \mathbf{\Omega})\},$$

$$\eta_i(\nu, \mathbf{\Omega}) = \sum_{nmr} \sum_{qq'} (-1)^{q+q'} \frac{8\pi^3 e_0^2}{ch\nu} \nu_{rm} \nu_{rn} \mathcal{N} \times$$

$$\times \mathrm{Re}\{(r_q)_{rm}(r_{q'})^*_{rn} \rho_{mn} \Phi(\nu_{rm} - \nu) \mathcal{T}_{-q-q'}(i, \mathbf{\Omega})\},$$

$$\rho_i(\nu, \mathbf{\Omega}) = \eta_i(\nu, \mathbf{\Omega}) \{\mathrm{Re} \to \mathrm{Im}\} \qquad (i = 1, 2, 3),$$

$$\eta_i^s(\nu, \mathbf{\Omega}) = \frac{c^2}{2h\nu^3} \varepsilon_i(\nu, \mathbf{\Omega}),$$

$$\rho_i^s(\nu, \mathbf{\Omega}) = \eta_i^s(\nu, \mathbf{\Omega}) \{\mathrm{Re} \to \mathrm{Im}\} \qquad (i = 1, 2, 3),$$

Landi Degl'Innocenti
(1983, Solar Physics 85,pp.3-31)

Tensors related with
Spherical tensors.

Density matrix

$$\rho_{mn}$$

Radiative Transfer

$$\frac{\mathrm{d}}{\mathrm{d}s}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \varepsilon_I \\ \varepsilon_Q \\ \varepsilon_U \\ \varepsilon_V \end{pmatrix} - \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} + \begin{pmatrix} \eta_I^s & \eta_Q^s & \eta_U^s & \eta_V^s \\ \eta_Q^s & \eta_I^s & \rho_V^s & -\rho_U^s \\ \eta_U^s & -\rho_V^s & \eta_I^s & \rho_Q^s \\ \eta_V^s & \rho_U^s & -\rho_Q^s & \eta_I^s \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

Spherical tensors
showing the
anisotropy
of the radiation .

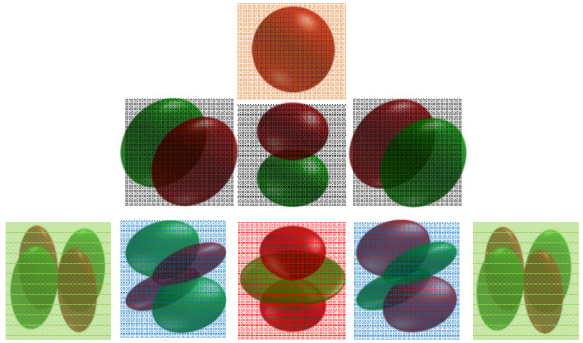
$$\bar{J}_Q^K$$

Tensors related with Spherical tensors and showing the effect by each Stokes components.

$$\mathcal{T}_{qq'}(0,\Omega) = \begin{bmatrix} \frac{1}{4}(1+\cos^2\theta) & -\frac{1}{2\sqrt{2}}\sin\theta\cos\theta\, e^{-i\chi} & \frac{1}{4}\sin^2\theta\, e^{-2i\chi} \\ -\frac{1}{2\sqrt{2}}\sin\theta\cos\theta\, e^{i\chi} & \frac{1}{2}\sin^2\theta & \frac{1}{2\sqrt{2}}\sin\theta\cos\theta\, e^{-i\chi} \\ \frac{1}{4}\sin^2\theta\, e^{2i\chi} & \frac{1}{2\sqrt{2}}\sin\theta\cos\theta\, e^{i\chi} & \frac{1}{4}(1+\cos^2\theta) \end{bmatrix}$$

?

Effect by Stokes-*I*



$$\mathcal{T}_{qq'}(1,\Omega) = \begin{bmatrix} -\frac{1}{4}\sin^2\theta\cos 2\gamma & \frac{1}{2\sqrt{2}}(-\sin\theta\cos\theta\cos 2\gamma + i\sin\theta\sin 2\gamma)\, e^{-i\chi} & -\frac{1}{4}[(1+\cos^2\theta)\cos 2\gamma + 2i\cos\theta\sin 2\gamma]\, e^{-2i\chi} \\ \frac{1}{2\sqrt{2}}(-\sin\theta\cos\theta\cos 2\gamma - i\sin\theta\sin 2\gamma)\, e^{i\chi} & \frac{1}{2}\sin^2\theta\cos 2\gamma & \frac{1}{2\sqrt{2}}(\sin\theta\cos\theta\cos 2\gamma - i\sin\theta\sin 2\gamma)\, e^{-i\chi} \\ -\frac{1}{4}[(1+\cos^2\theta)\cos 2\gamma - 2i\cos\theta\sin 2\gamma]\, e^{2i\chi} & \frac{1}{2\sqrt{2}}(\sin\theta\cos\theta\cos 2\gamma + i\sin\theta\sin 2\gamma)\, e^{i\chi} & -\frac{1}{4}\sin^2\theta\cos 2\gamma \end{bmatrix}$$

Effect by Stokes-*Q*

$$\mathcal{T}_{qq'}(2,\Omega) = \begin{bmatrix} \frac{1}{4}\sin^2\theta\sin 2\gamma & \frac{1}{2\sqrt{2}}(\sin\theta\cos\theta\sin 2\gamma + i\sin\theta\cos 2\gamma)\, e^{-i\chi} & \frac{1}{4}[(1+\cos^2\theta)\sin 2\gamma + 2i\cos\theta\cos 2\gamma]\, e^{-2i\chi} \\ \frac{1}{2\sqrt{2}}(\sin\theta\cos\theta\sin 2\gamma - i\sin\theta\cos 2\gamma)\, e^{i\chi} & -\frac{1}{2}\sin^2\theta\cos 2\gamma & \frac{1}{2\sqrt{2}}(-\sin\theta\cos\theta\sin 2\gamma - i\sin\theta\cos 2\gamma)\, e^{-i\chi} \\ \frac{1}{4}[(1+\cos^2\theta)\sin 2\gamma - 2i\cos\theta\cos 2\gamma]\, e^{2i\chi} & \frac{1}{2\sqrt{2}}(-\sin\theta\cos\theta\sin 2\gamma + i\sin\theta\cos 2\gamma)\, e^{i\chi} & \frac{1}{4}\sin^2\theta\sin 2\gamma \end{bmatrix}$$

Effect by Stokes-*U*

$$\mathcal{T}_{qq'}(3,\Omega) = \begin{bmatrix} -\frac{1}{2}\cos\theta & \frac{1}{2\sqrt{2}}\sin\theta\, e^{-i\chi} & 0 \\ \frac{1}{2\sqrt{2}}\sin\theta\, e^{i\chi} & 0 & \frac{1}{2\sqrt{2}}\sin\theta\, e^{-i\chi} \\ 0 & \frac{1}{2\sqrt{2}}\sin\theta\, e^{i\chi} & \frac{1}{2}\cos\theta \end{bmatrix}$$

Effect by Stokes-*V*

LandiDegl'Innocenti (1983, Solar Physics 85,pp.3-31)

The unbalance between horizontal and vertical is important.

- Not between outgoing and incoming.

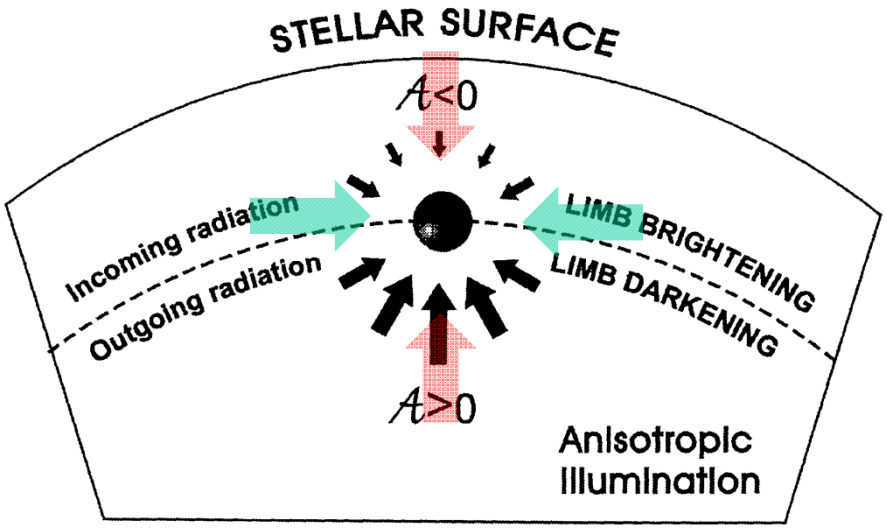
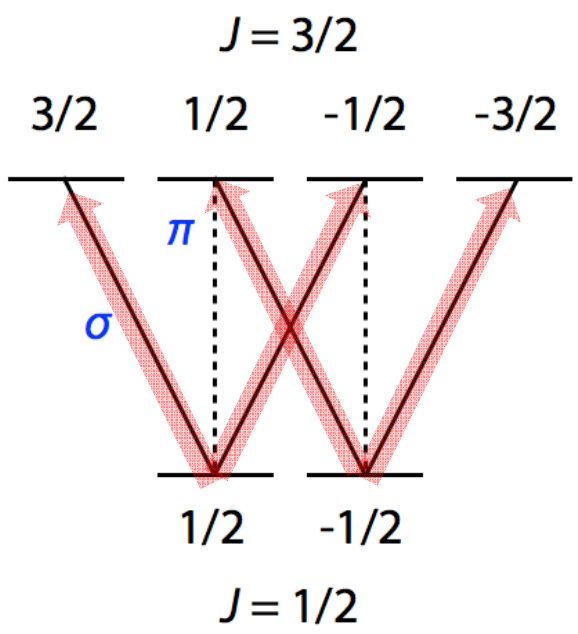
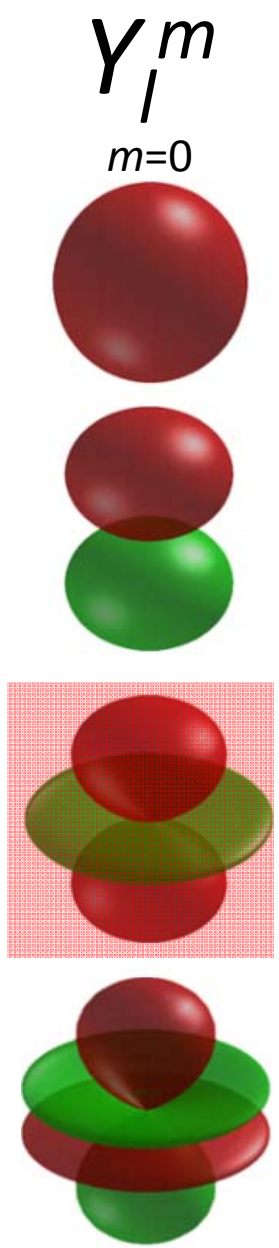
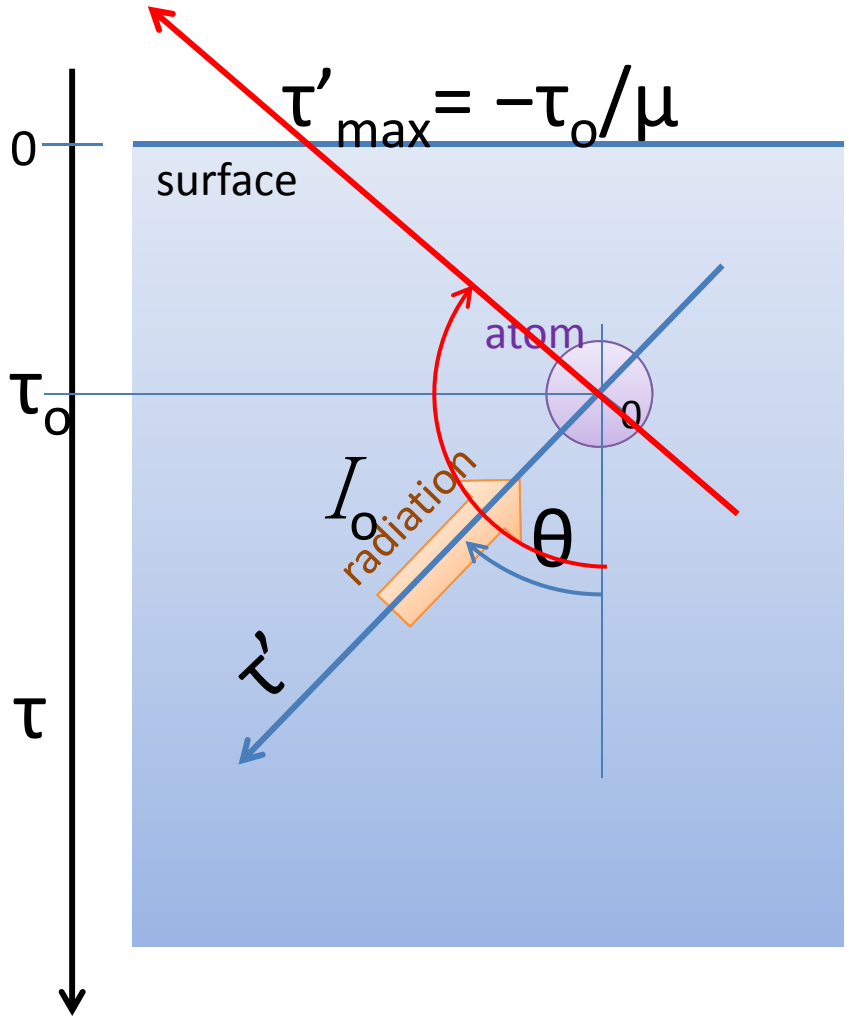


Figure 5. The anisotropic illumination in a stellar atmosphere.





Anisotropy in an unmagnetized Milne-Eddington atmosphere



- Q=U=V=0

$$\frac{dI}{d\tau'} = I - S$$

$$S \equiv a + b\tau$$

$$= a + b(\tau_o + \tau'\mu)$$

- Case: $\mu \equiv \cos\theta > 0$

$$I_o = a + b(\tau_o + \mu)$$

- Case: $\mu \equiv \cos\theta < 0$

$$I_o = a + b(\tau_o + \mu)$$

$$\underbrace{-(a + b\mu)e^{\tau_o/\mu}}_{\text{Surface effect}}$$

Anisotropy in an unmagnetized Milne-Eddington atmosphere

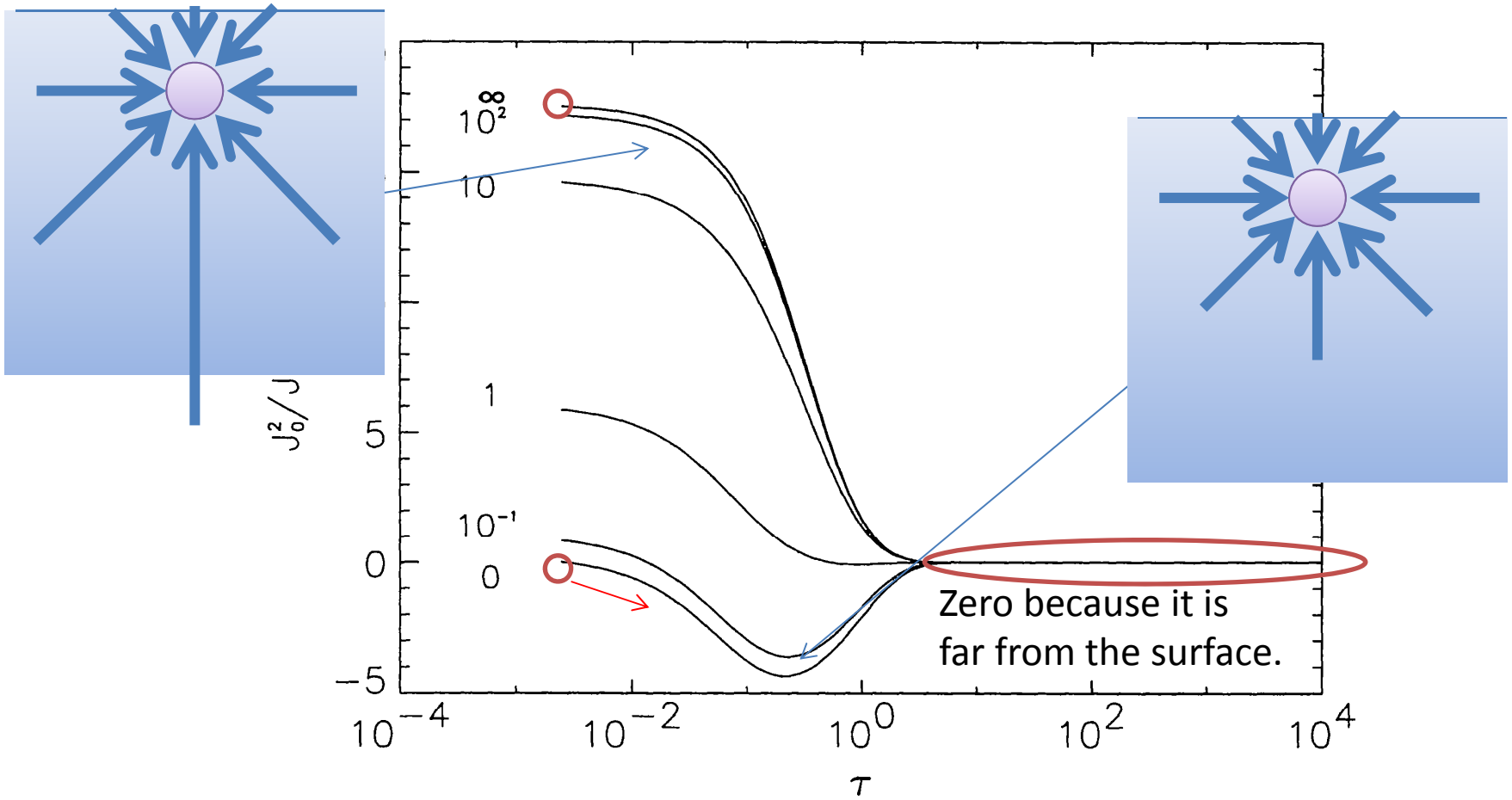
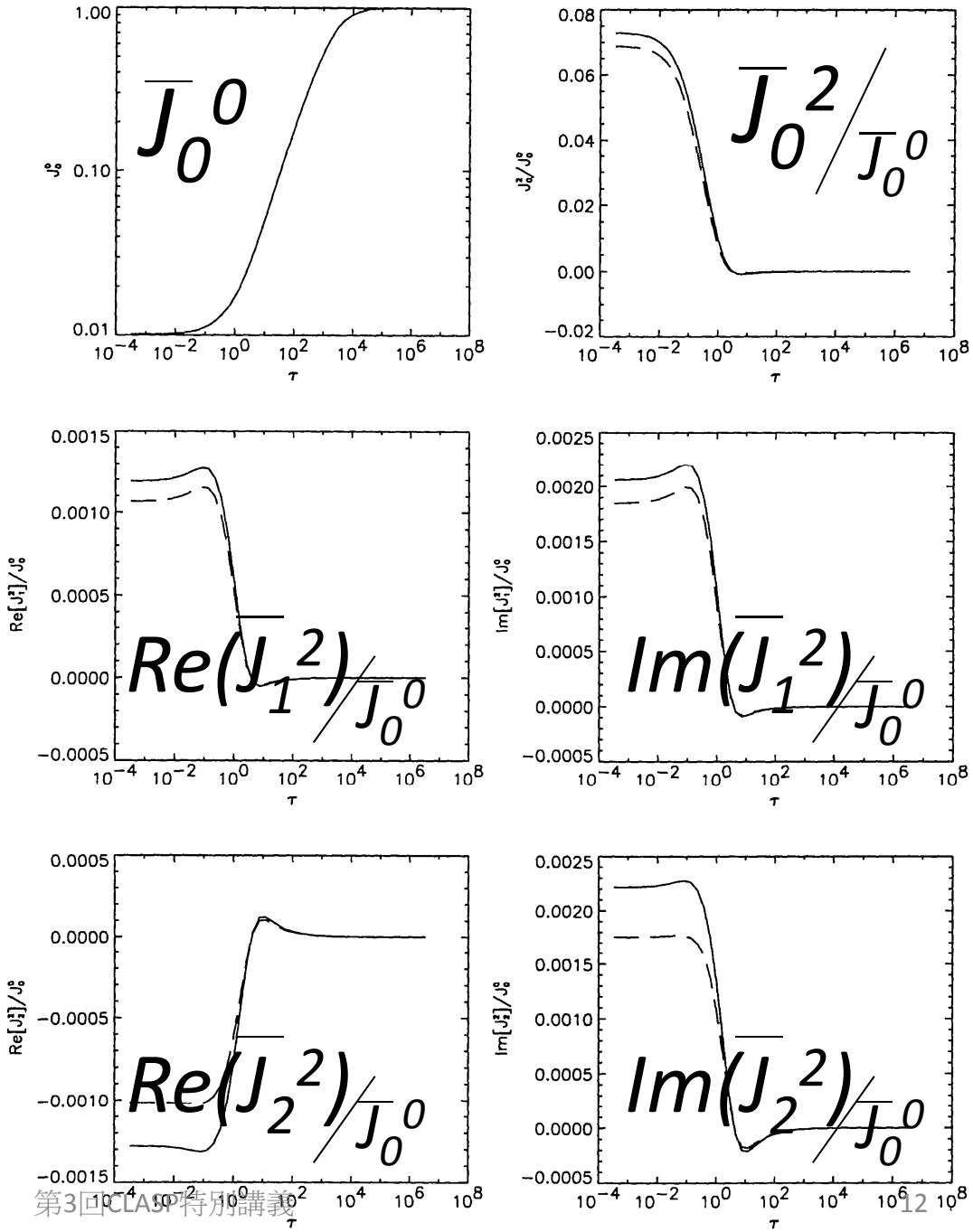


Figure 4. The anisotropy factor in Milne–Eddington atmospheres.

$\bar{J}_1^2$ & $\bar{J}_2^2$

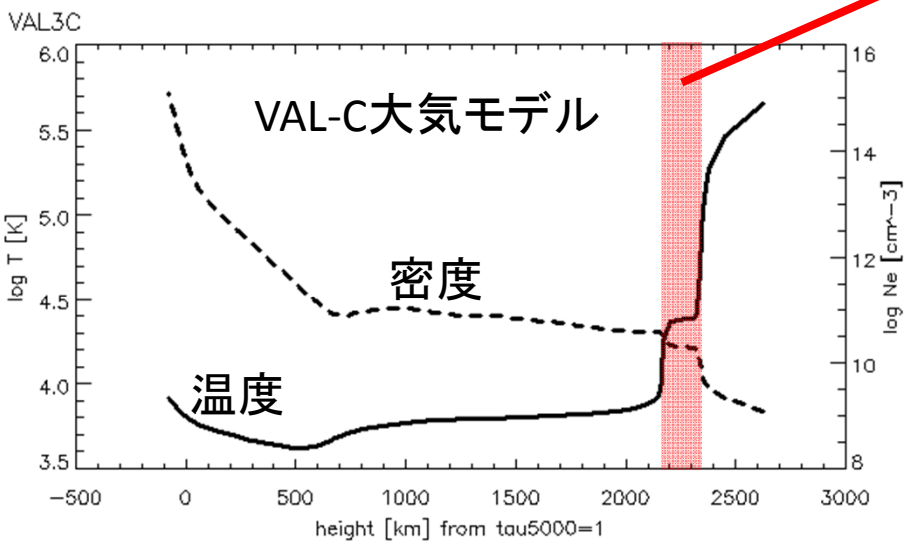
- 軸対称が崩れると J_1^2 & J_2^2も0でない。
- 水平磁場の例。
  - 等温大気を仮定。
  - dashはStokes-Iのみを考慮。

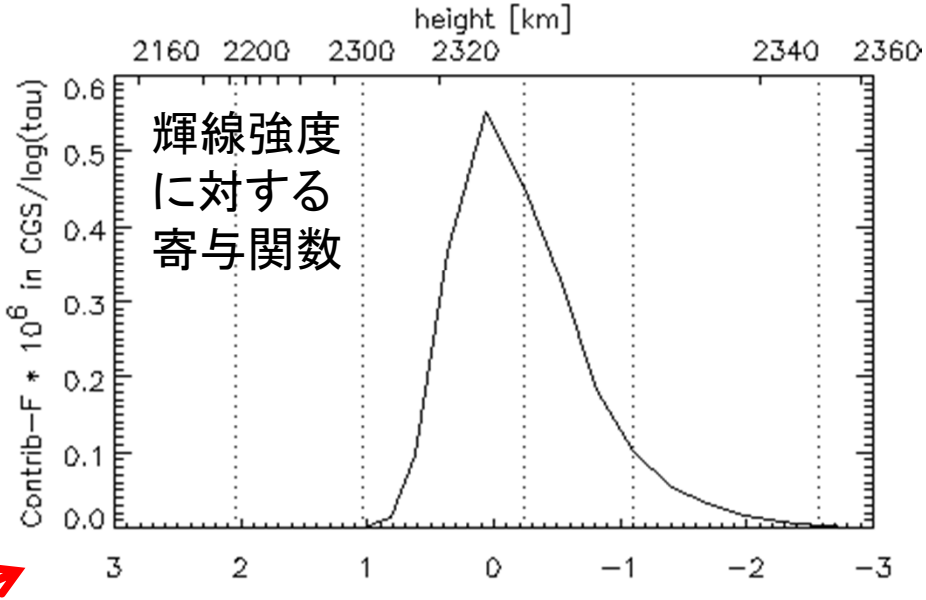


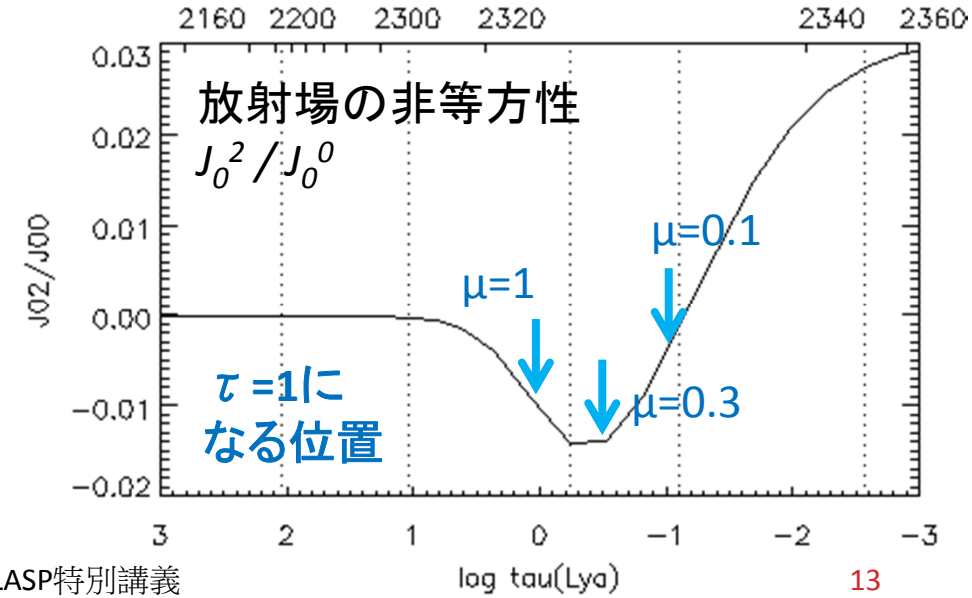
Trujillo Bueno
(2001, ASP Conf. 236, pp.161-195)

VAL-C大気モデル&MULTIによる計算例

- Lyα輝線は彩層上部～遷移層下部にて形成される。
- 輝線形成領域には、数%程度の非等方放射場が存在する。





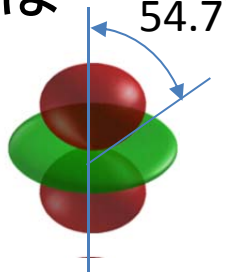


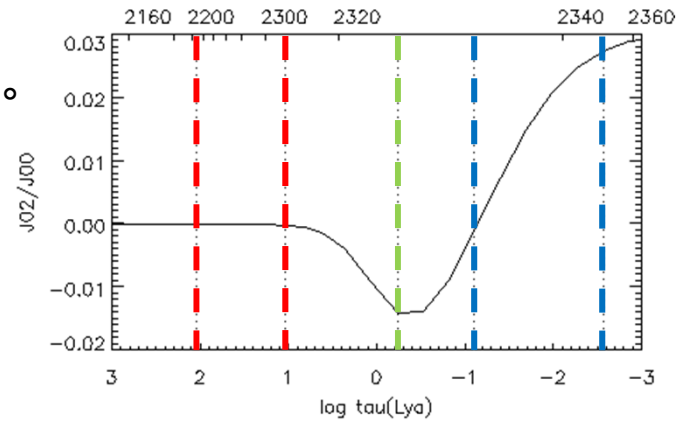
Lyα線での非等方放射場

Hanle効果に関連する “非等方性” は

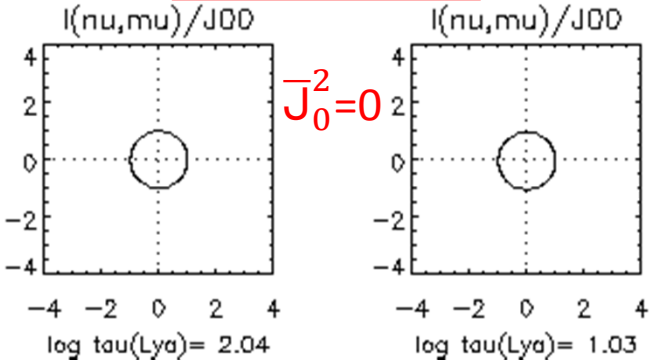
×: Incoming vs. Outgoing

○: 鉛直方向 vs. 水平方向



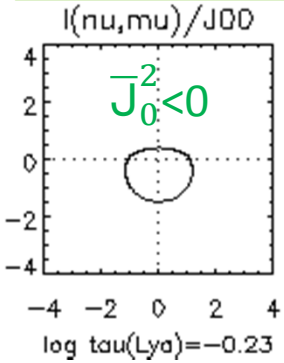


$\log \tau_{Ly\alpha} > 1$



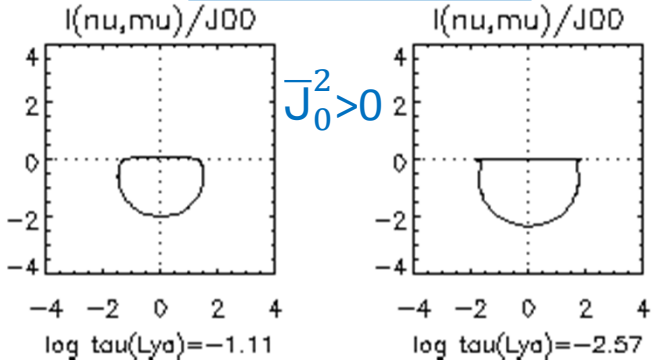
Incoming: 等方的
Outgoing: 等方的

$\log \tau_{Ly\alpha} \sim 0$



Incoming: 鉛直成分減少
Outgoing: まだ等方的

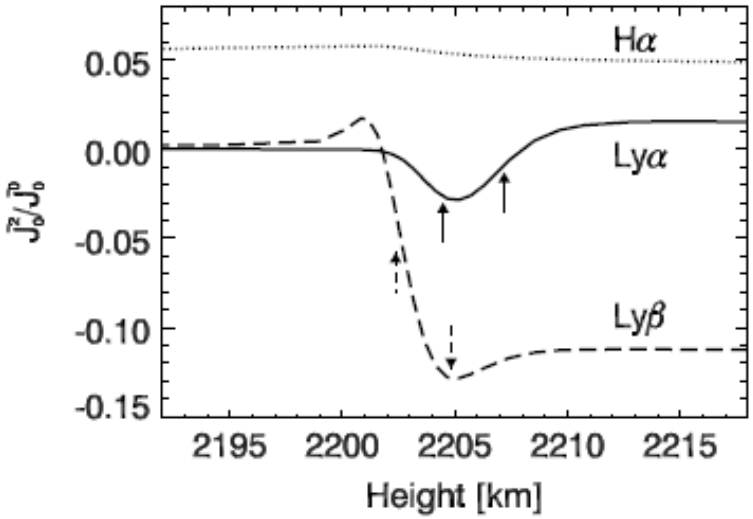
$\log \tau_{Ly\alpha} < -1$

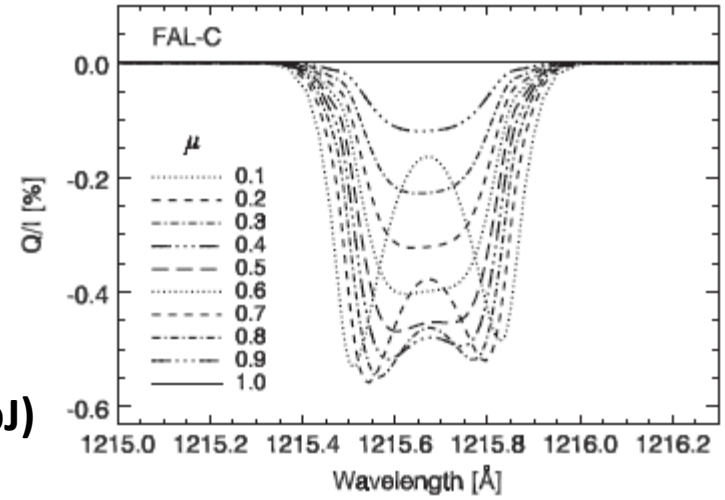


Incoming: ほぼ0
Outgoing: 水平成分減少

磁場がない場合のLyα線の直線偏光

- 非等方放射場 J_0^2による atomic polarizationにより、磁場がなくとも偏光が発生する。
- μ=1(中心)
  対称性により直線偏光はでない。
- μ=0.3～0.4
  非等方放射場の最大のところで τ =1となるので、最も強く直線偏光が発生する。
- μ=0.1
  既に非等方放射場が小さくなったところで(波長中心の) τ =1となるので、波長中心の直線偏光は弱い。

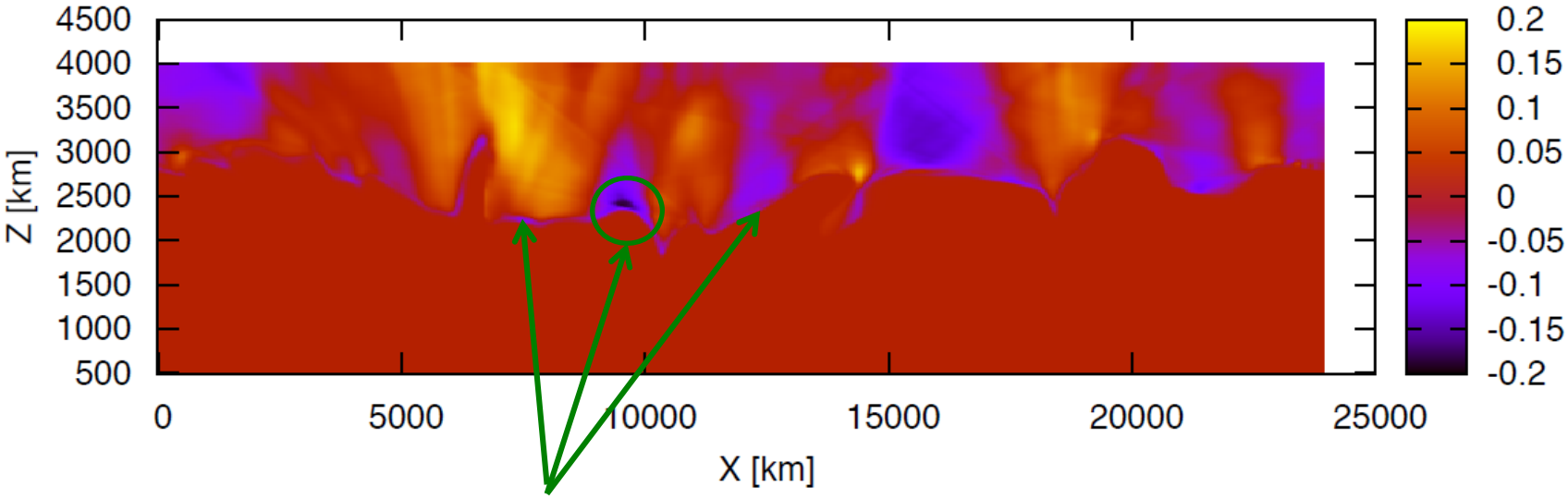




Trujillo Bueno et al. (2011, ApJ)

3次元シミュレーション太陽大気(Carlsson et al.)に基づくLyα輝線の放射場の非等方性

Truiillo Bueno et al. (2011, private communication)



- 正にLyα輝線を放つ(彩層上部～)遷移層において非等方放射場が卓越している。