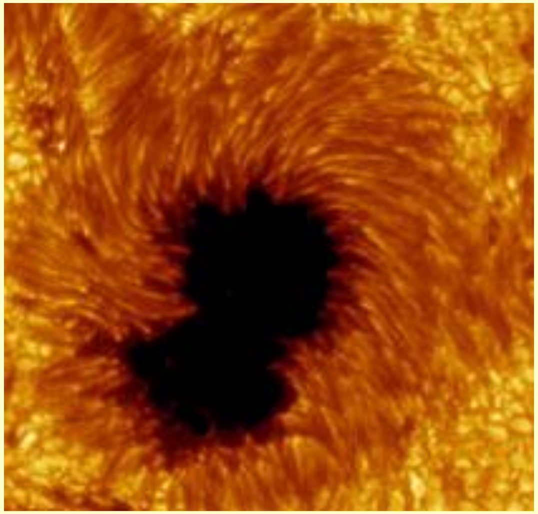
The Flux Transport Dynamo, Flux Tubes and Helicity



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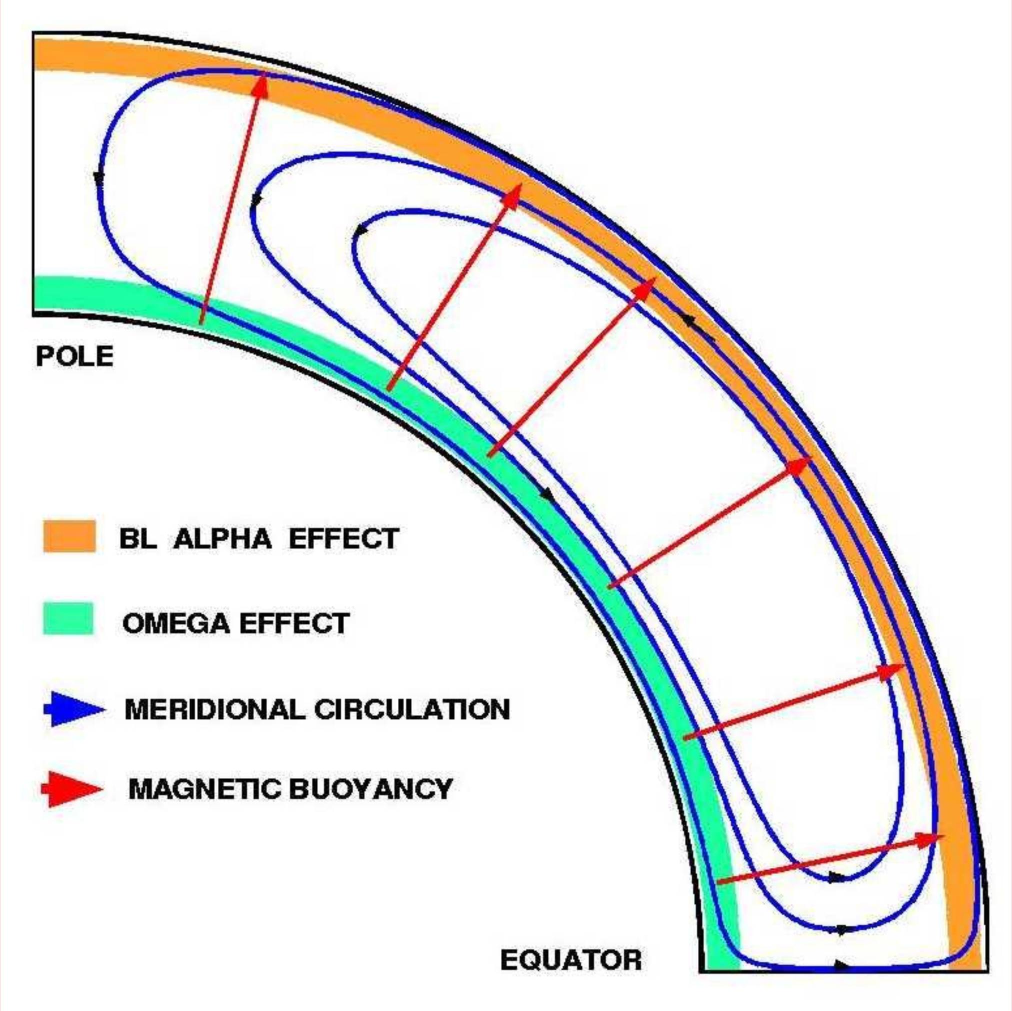
Flux transport dynamo in the Sun (Choudhuri, Schussler & Dikpati 1995; Durney 1995)

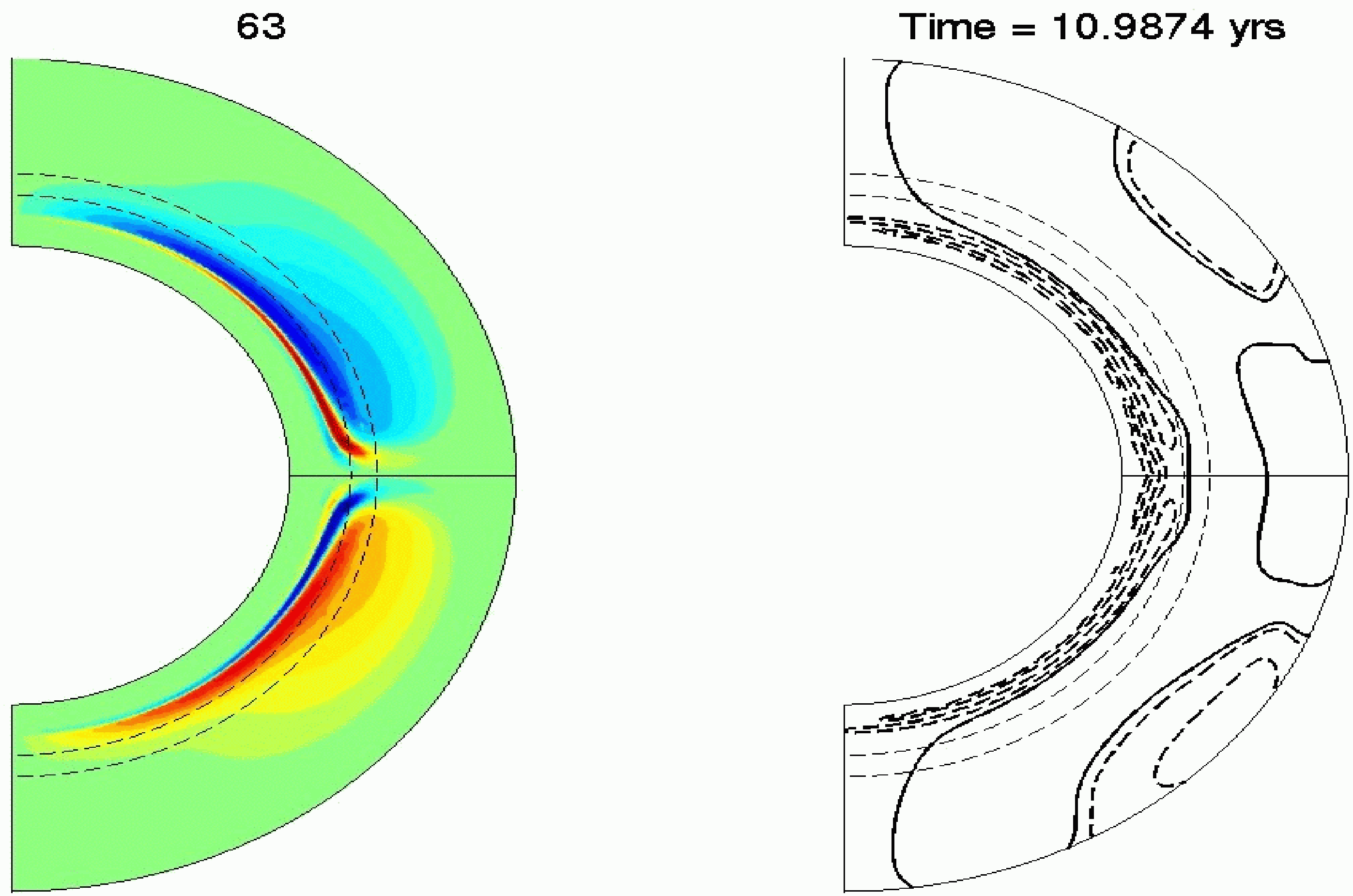
Differential rotation > toroidal field generation

Babcock-Leighton process > poloidal field generation

Meridional circulation carries toroidal field equatorward & poloidal field poleward

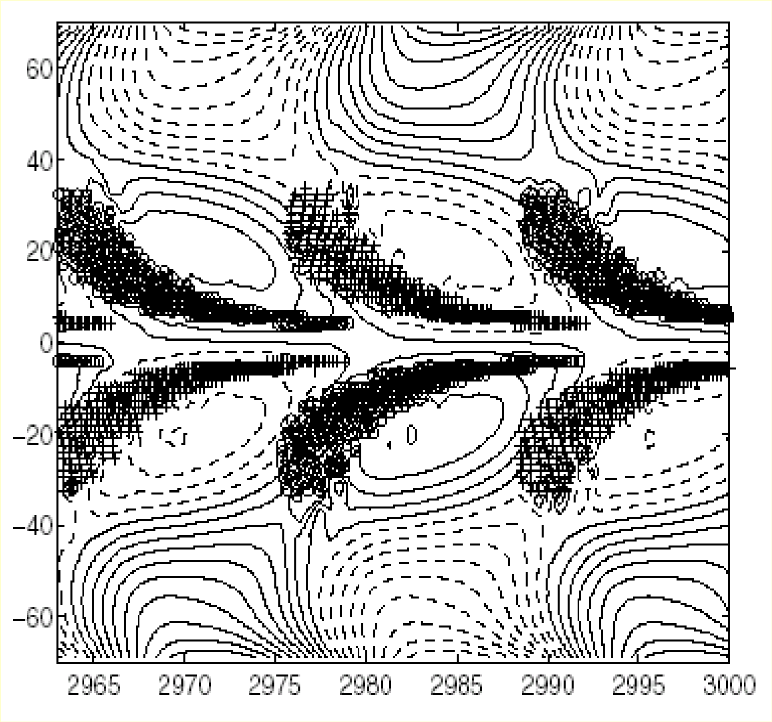


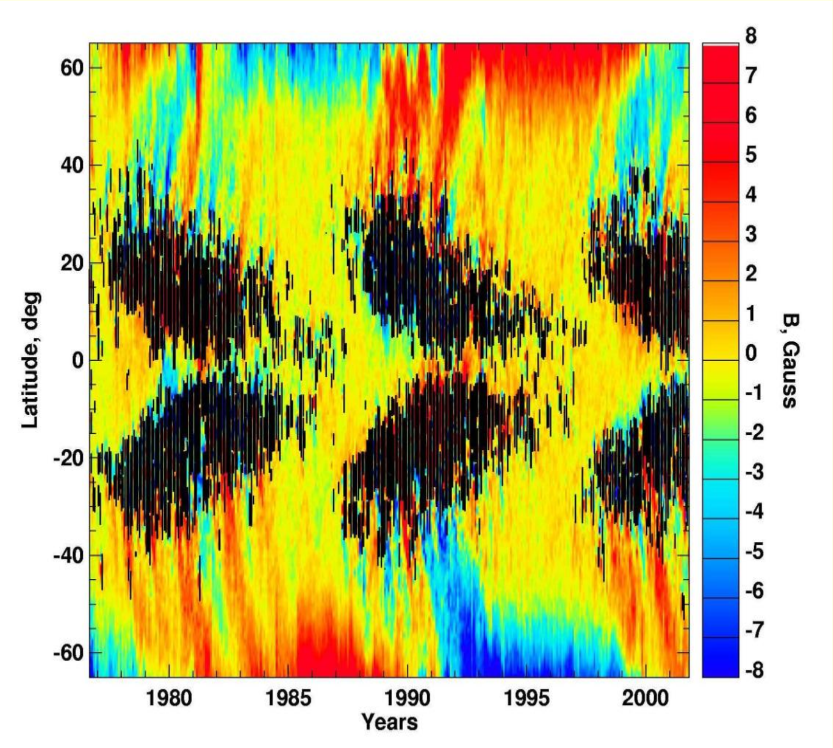
Basic idea was given by Wang, Sheeley & Nash (1991)


63
Time = 10.9874 yrs

Results from detailed model of Chatterjee, Nandy & Choudhuri (2004)

Butterfly diagrams with both sunspot eruptions and weak field at the surface > Reasonable fit between theory & observation





Basic Equations

Magnetic field

$$\boldsymbol{B} = B(r,\theta)\boldsymbol{e}_\phi + \nabla \times [A(r,\theta)\boldsymbol{e}_\phi],$$

Velocity field

$$\Omega(r,\theta)\, r\sin\theta\, \mathbf{e}_\phi + \mathbf{v}$$

$$\frac{\partial A}{\partial t} + \frac{1}{s}(\boldsymbol{v}.\nabla)(sA) = \eta_{\rm p}\left(\nabla^2 - \frac{1}{s^2}\right)A + \alpha B,$$

$$\frac{\partial B}{\partial t} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_r B) + \frac{\partial}{\partial \theta}(v_\theta B)\right] = \eta_{\rm t}\left(\nabla^2 - \frac{1}{s^2}\right)B + s(\boldsymbol{B}_{\rm p}.\nabla)\Omega + \frac{1}{r}\frac{{\rm d}\eta_{\rm t}}{{\rm d}r}\frac{\partial}{\partial r}(rB)$$

The code *Surya* solves these equations

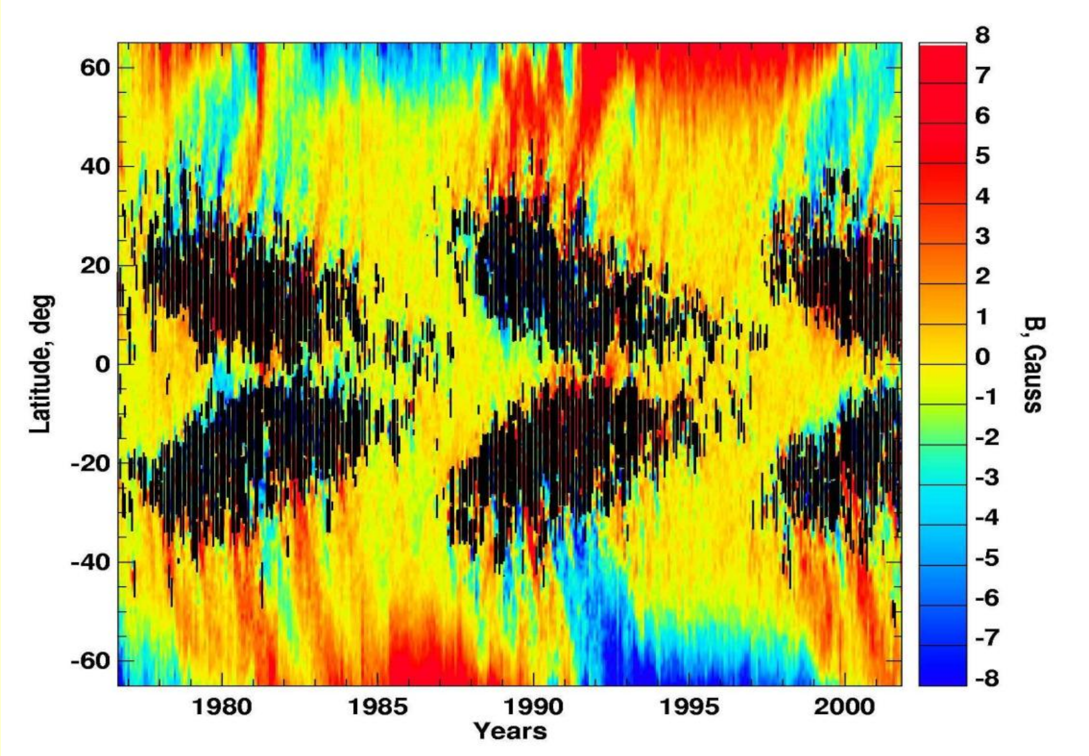
For a range of parameters, the code relaxes to periodic solutions (Nandy & Choudhuri 2002)

Parameters to be specified

- Differential rotation Ω (provided by helioseismology)
- Meridional circulation (provided by helioseismology till depth $0.85R$)
- Poloidal field source parameter α (BL process observed on the surface, but below the surface?)
- Turbulent diffusivities η_p and η_t (surface values estimated, reasonable assumptions underneath)
- Magnetic buoyancy

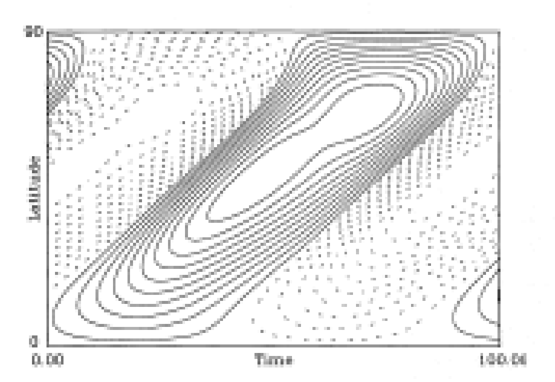
Major uncertainties at the present time : (i) Penetration depth of meridional circulation; (ii) Distribution of α below the surface; (iii) Most satisfactory way of treating magnetic buoyancy.

Constraints on parameters from observations

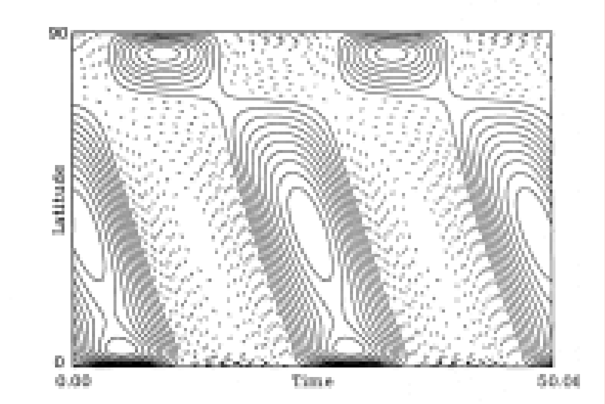


- Cyclic behaviour with a period of about 22 yrs
- The butterfly diagram of sunspots is restricted to low latitudes (below 40)
- The weak fields outside active regions drift poleward
- Polar field reversal takes place at the time of sunspot maximum
- The solar magnetic field appears dipolar (was it always so?)
- Magnetic helicity tends to be negative (positive) in northern (southern) hemisphere

From Choudhuri, Schussler & Dikpati (1995)



Without meridional circulation



With meridional circulation

Important time scales in the dynamo problem

- T_{tach} – Diffusion time scale in the tachocline
- T_{conv} – Diffusion time scale in convection zone
- T_{circ} – Meridional circulation time scale

$\mathbf{T_{tach} > T_{circ} > T_{conv}}$ **:** Choudhuri, Nandy, Chatterjee, Jiang, Karak, Hotta, Munoz-Jaramillo ...

$\mathbf{T_{tach} > T_{conv} > T_{circ}}$: Dikpati, Charbonneau, Gilman, de Toma…

Flux Transport dynamo

(Choudhuri, Schussler & Dikpati 1995)

High diffusivity model

(diffusion time ~ 5 yrs)

IISc group

(Choudhuri, Nandy, Chatterjee, Jiang, Karak)

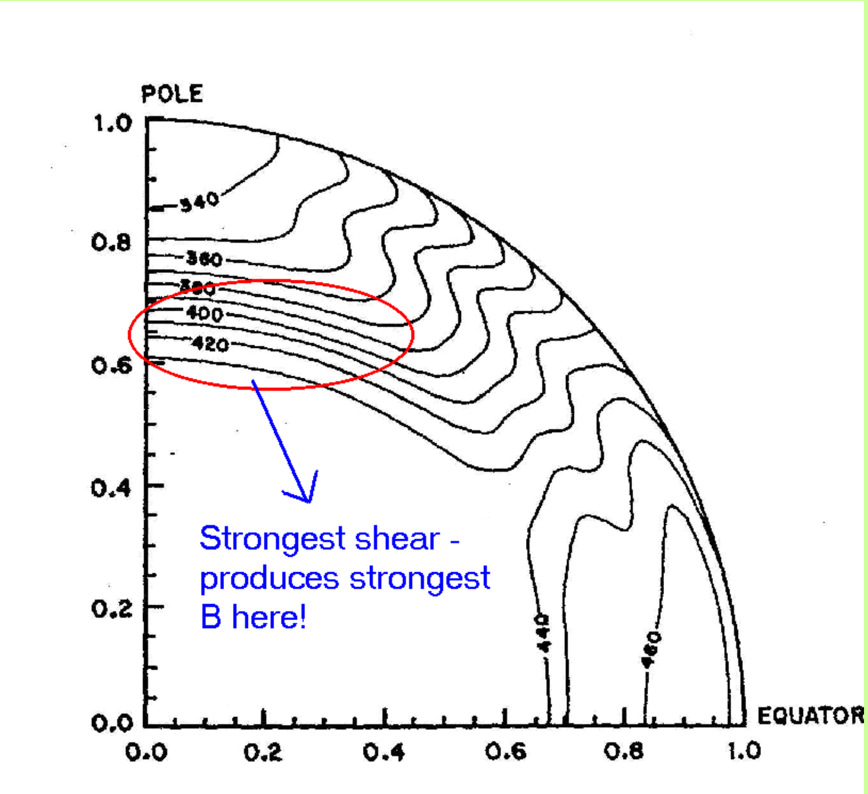
Low diffusivity model

(diffusion time ~ 200 yrs)

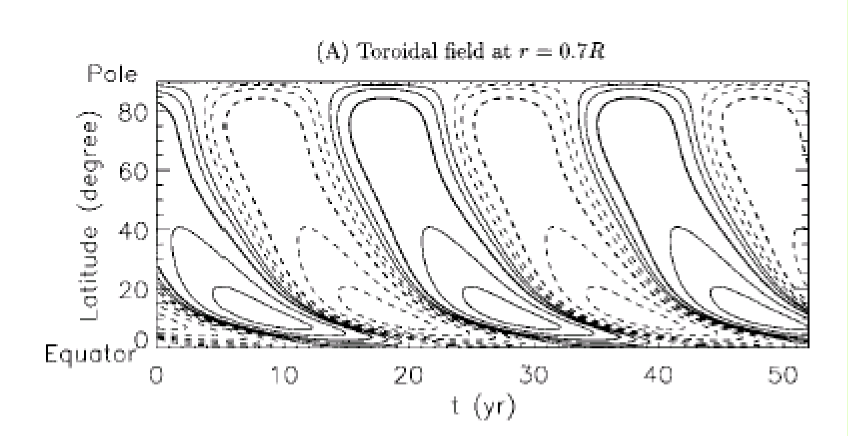
HAO group

(Dikpati, Charbonneau, Gilman, de Toma)

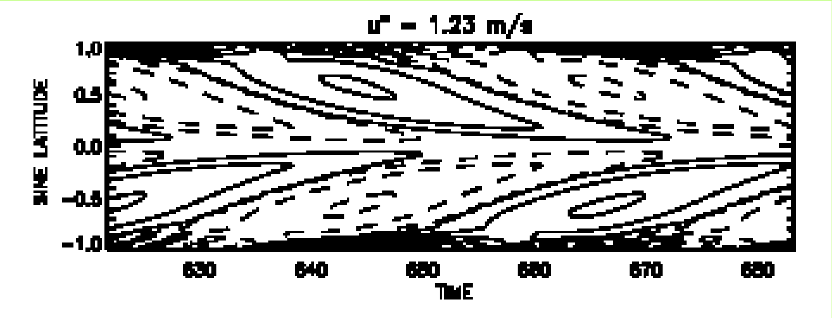
Differences between these models were systematically studied by Jiang, Chatterjee & Choudhuri (2007) and Yeates, Nandy & Mckay (2008)



Theoretical butterfly diagrams



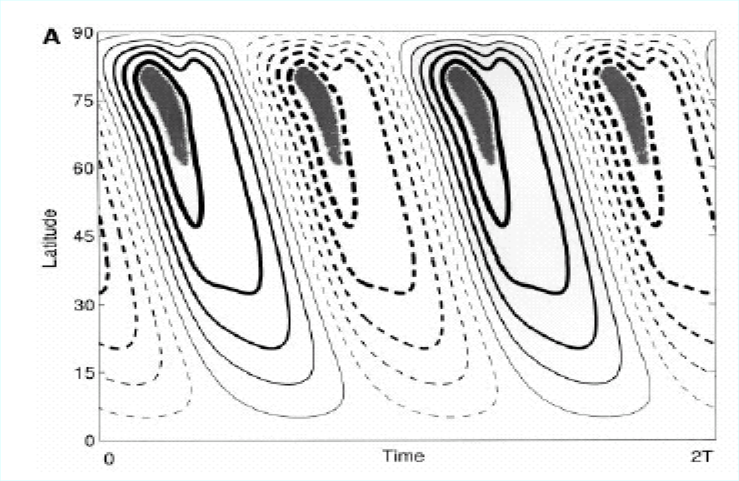
Dikpati & Charbonneau 1999



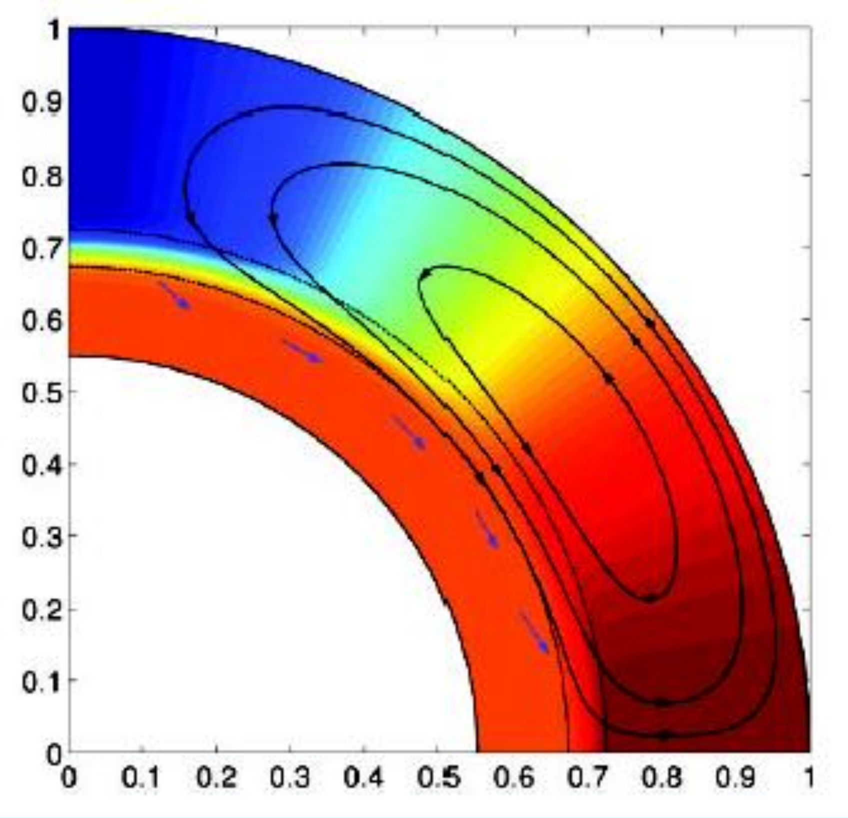
Kuker, Rudiger & Schultz 2001

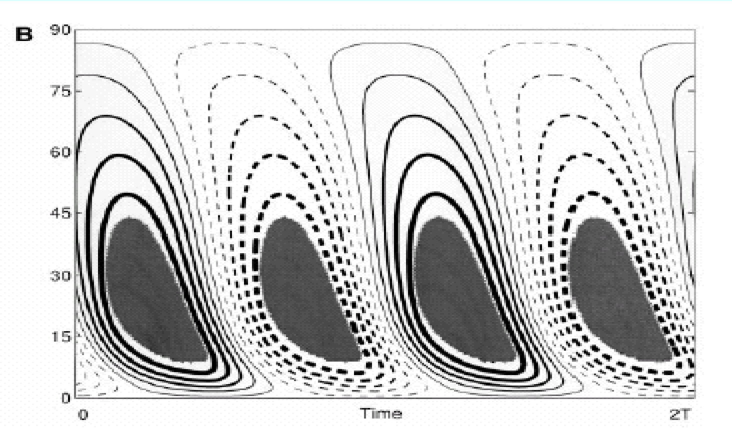
Solar-like rotation tends to produce sunspots at high latitudes > Non-realistic butterfly diagrams

Nandy & Choudhuri (2002) introduced meridional flow penetrating slightly below the tachocline to produce sunspots at correct latitudes



Without penetrating flow

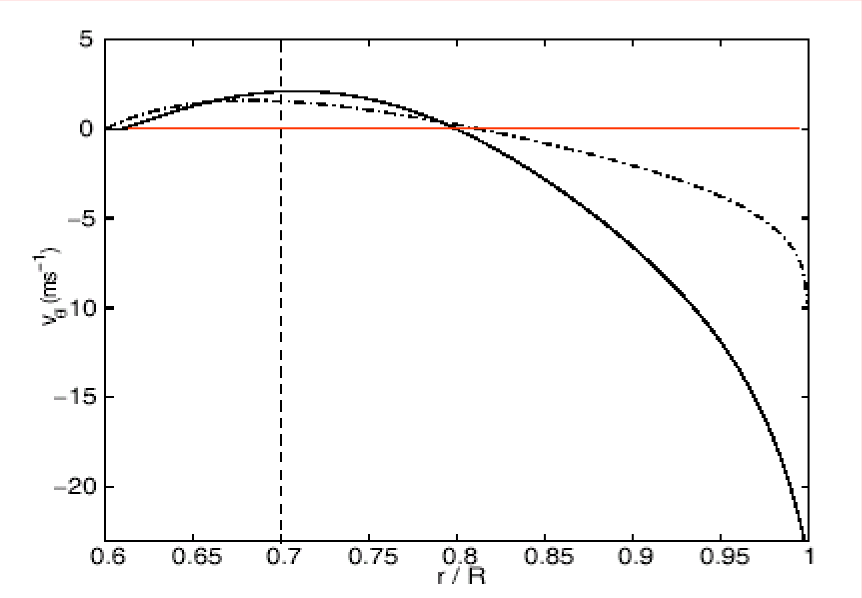




With penetrating flow

We believe that the meridional circulation has to penetrate slightly below tachocline, but HAO group claim that this is not possible!

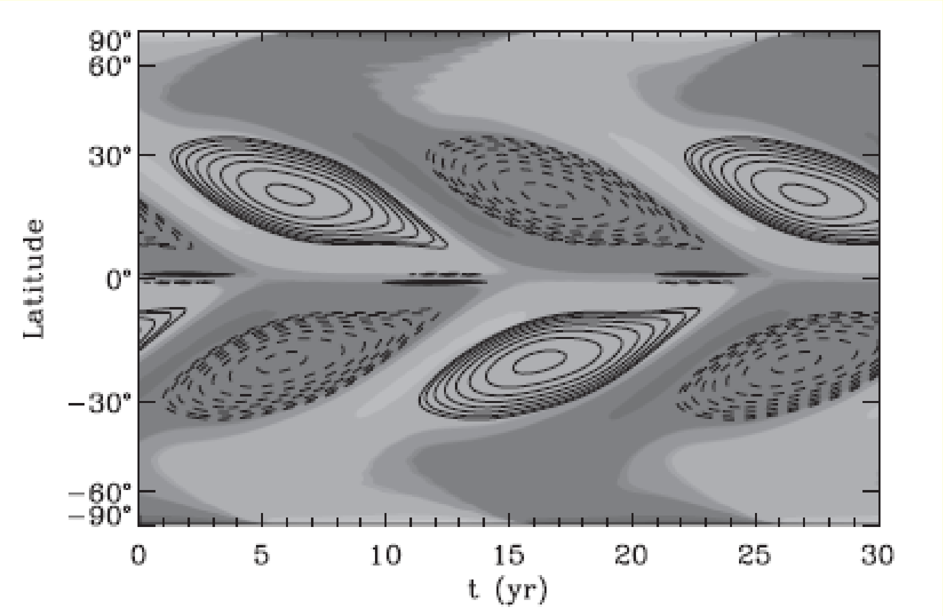
The tachocline is the least understood region of the Sun



v_θ as function of depth from our model (solid) and from Dikpati & Charbonneau 1999 (dotted)

Gilman & Miesch (2004) argued against the penetration of meriodional circulation below convection zone, whereas Garaud & Brummel (2008) found their argument to be flawed

Dikpati et al. (2004) claim that they can produce good results with non-penetrating circulation, but physics details are not clear – no mention of magnetic buoyancy in the paper!!!



No other group could reprocduce their result

Dikpati & Gilman (2008) wrote a strange paper on their method

Distribution of α-coefficient (source of poloidal field)

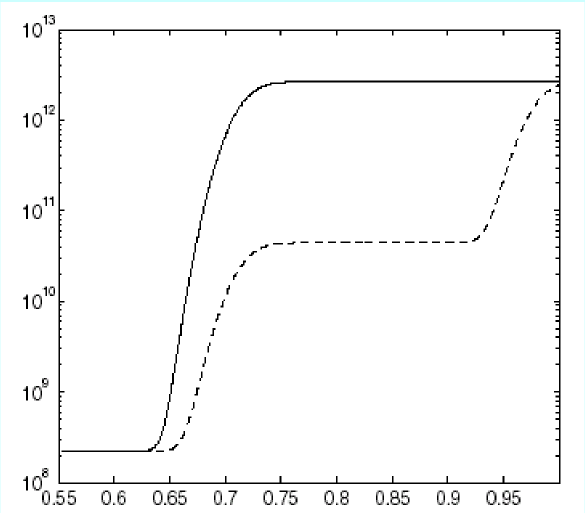
We see the generation of poloidal field at the solar surface

Dikpati & Gilman (2001) and Bonanno et al. (2002) claim that α at the surface alone would produce quadrupolar parity and argue for α in the interior

Chatterjee, Nandy & Choudhuri (2004) show that even surface α can produce dipolar parity with suitable choice of turbulent diffusion

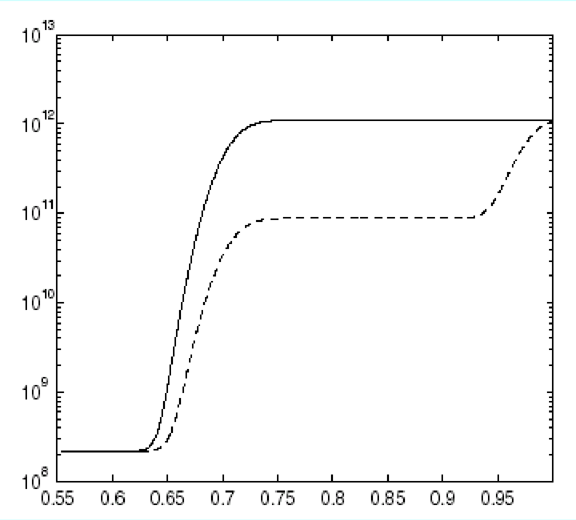
But we presumably need an α different from Babcock-Leighton α to pull the dynamo out of grand minima!

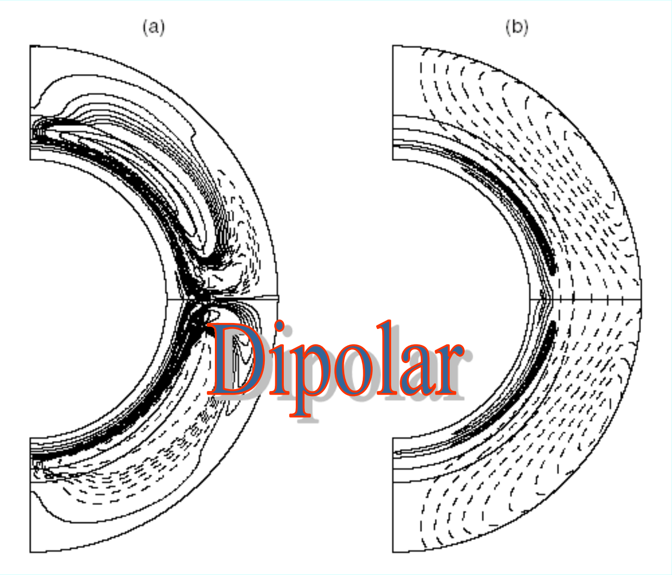
From **Chatterjee, Nandy & Choudhuri (2004)**

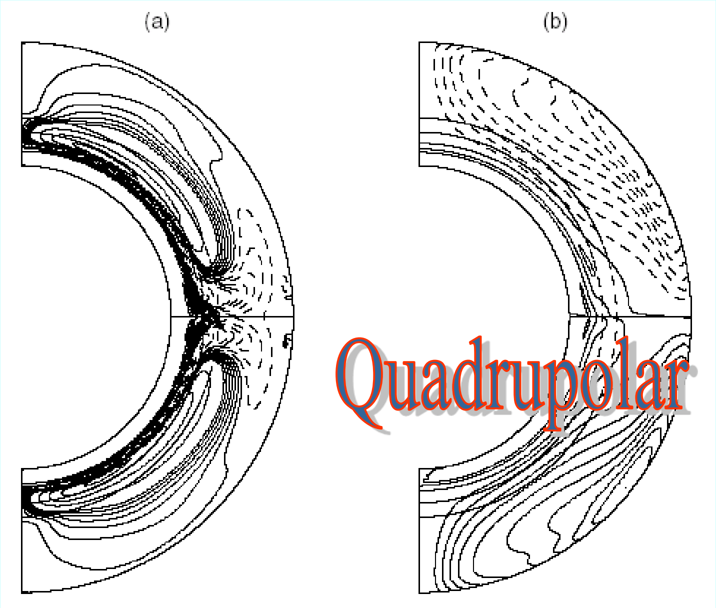


Plots of η_p (solid) and η_t (dashed) as functions of depth

(a) Toroidal (b) Poloidal





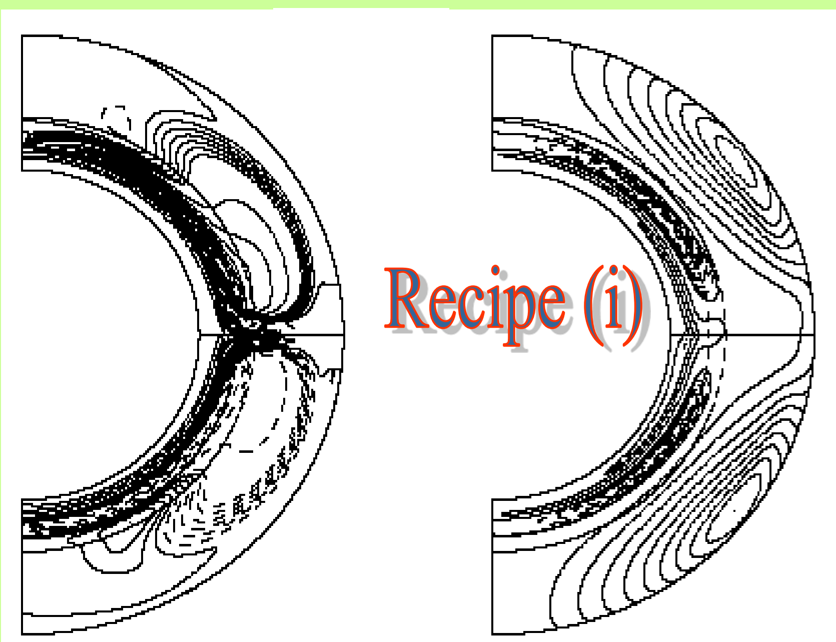


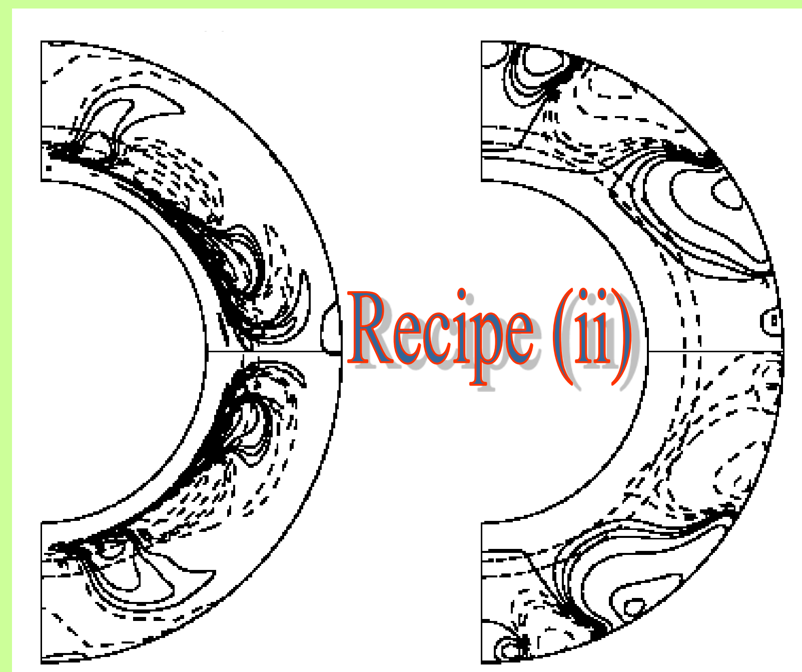
Hotta & Yokoyama (2010) also obtained dipolar parity by making the diffusivity large near the surface.

Two popular recipes for treating magnetic bouyancy

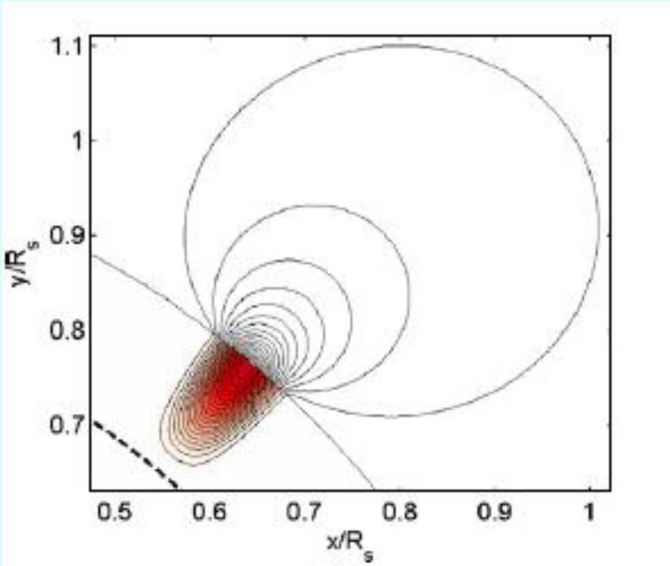
(i) If *B* inside convection zone is larger than a critical value, move a part of it to the surface – we follow this

(ii) In poloidal field source at surface, put value of *B* from bottom of convection zone – HAO group follow this (first proposed by Choudhuri & Dikpati 1999)





Munoz-Jaramillo et al. (2011) suggest that the double ring method originally proposed by Durney (1995) is the best method for treating magnetic buoyancy



(b)

Magnetic buoyancy is essentially a 3D process and cannot be treated adequately in a 2D model – main source of uncertainly in 2D flux transport dynamo models.

Should we go for 3D kinematic models?

Parameters to be specified

- Differential rotation Ω (provided by helioseismology)
- Meridional circulation (provided by helioseismology till depth 0.85*R*)
- Poloidal field source parameter α (BL process observed on the surface, but below the surface?)
- Turbulent diffusivities η_p and η_t (surface values estimated, reasonable assumptions underneath)
- Magnetic buoyancy

Major uncertainties at the present time : (i) Penetration depth of meridional circulation; (ii) Distribution of α below the surface; (iii) Most satisfactory way of treating magnetic buoyancy.

Dynamo equations just give you information about mean fields
You need to use additional physics to study flux tubes.

How are 10^5 G flux tubes produced in the tachocline? (Choudhuri 2003)
Toroidal field generation equation:

$$\frac{\partial B_\phi}{\partial t} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_r B_\phi) + \frac{\partial}{\partial \theta}(v_\theta B_\phi)\right] = \eta\left(\nabla^2 - \frac{1}{s^2}\right) B_\phi + \\ + s(\mathbf{B}_p.\nabla)\Omega - \nabla\eta \times \left(\nabla \times B_\phi \mathbf{e}_\phi\right),$$

Toroidal field generated

$$B_\phi \approx s(\mathbf{B}_p \cdot \nabla)\Omega\,\tau,$$

From which

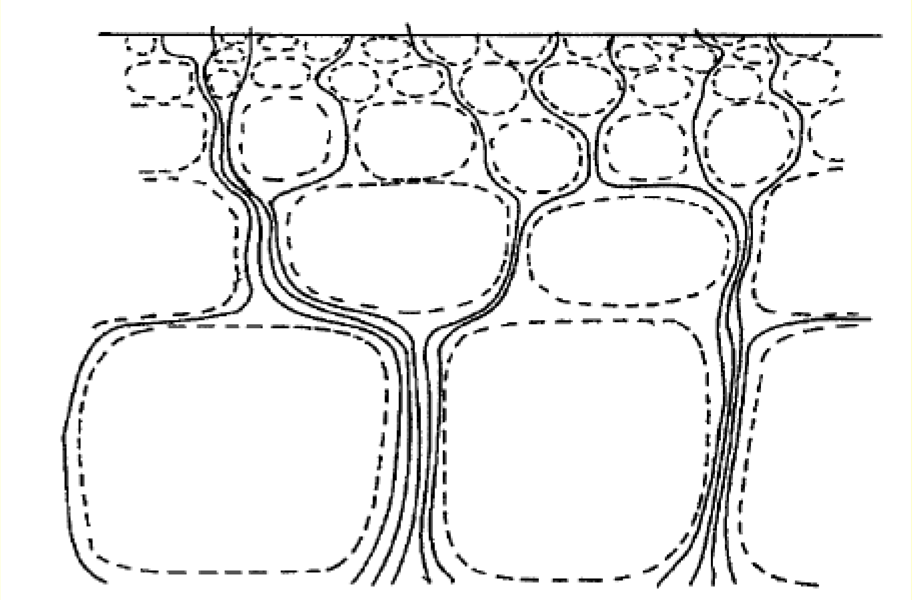
$$\frac{B_\phi}{B_r} \approx s\frac{\Delta\Omega}{\Delta r}\tau.$$

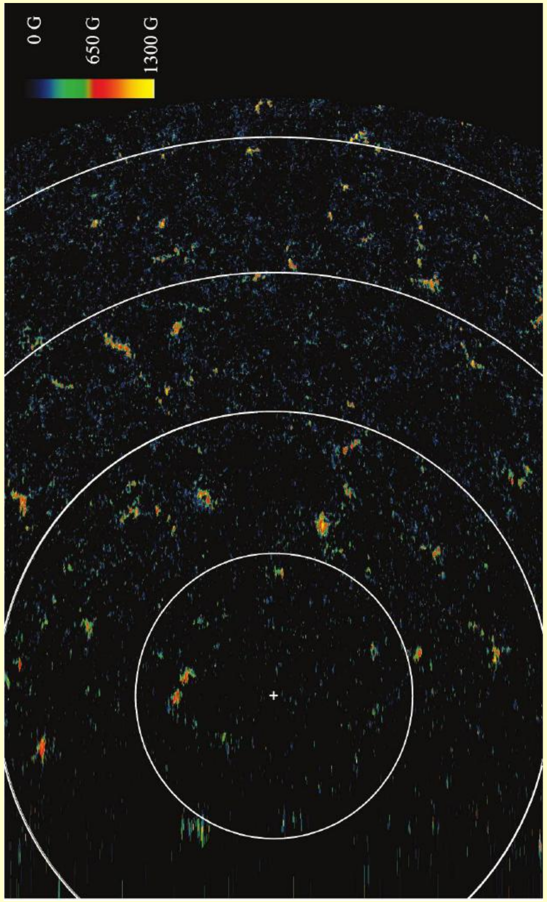
Order of magnitude

$$\frac{B_\phi}{B_r} \approx 1000.$$

You need to start from at least a few hundred G poloidal field to create a 10^5 G flux tube

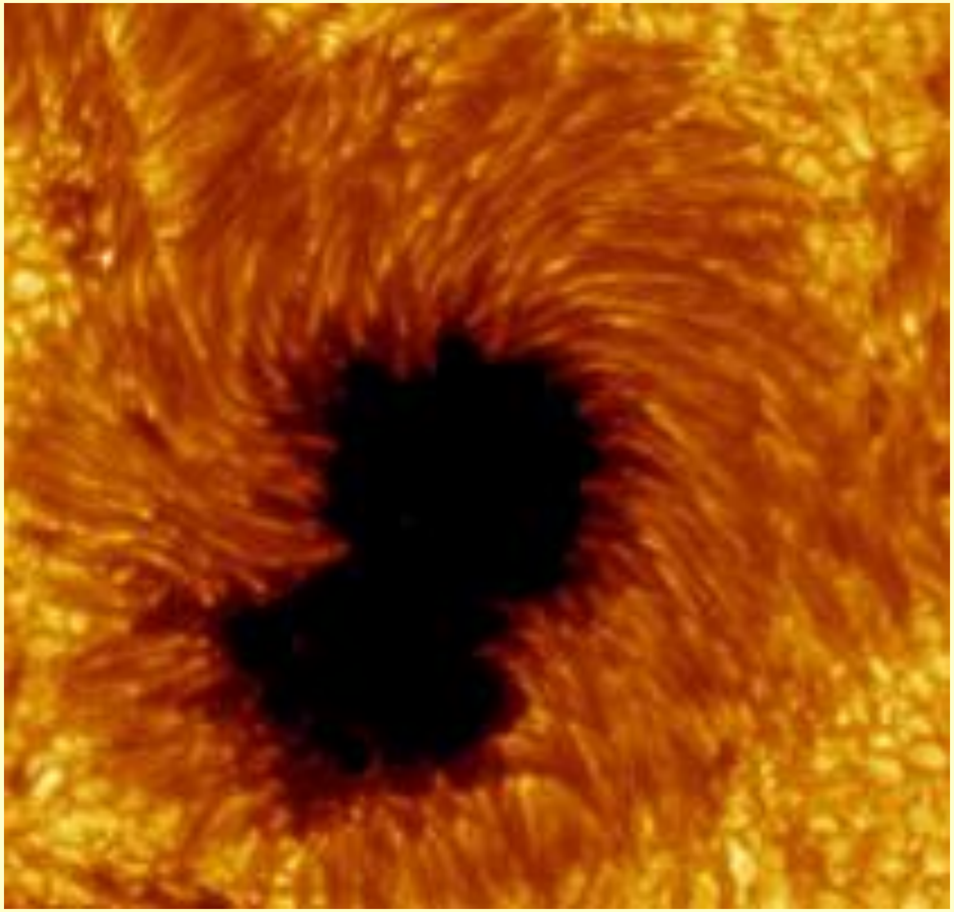
From Choudhuri (2003)





Hinode discovered such flux concentrations (Tsuneta et al. 2008)

Many sunspots appear twisted



Hale 1927;
Richardson 1941 –

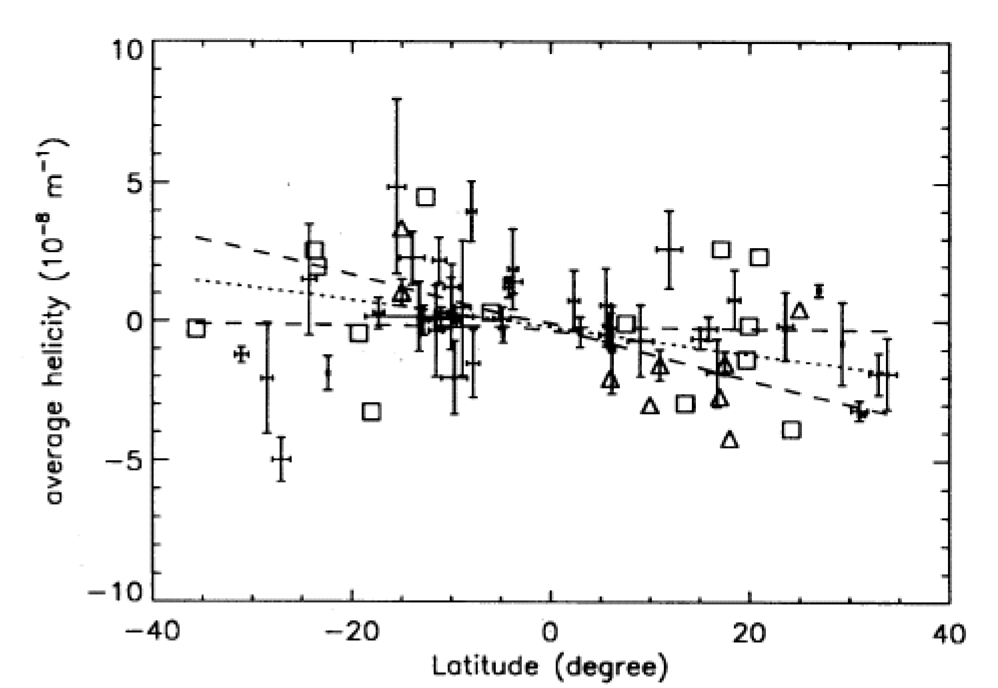
Left-handed in northern hemisphere and right-handed in southern

Current along the axis of the sunspot

Vector magnetogram measurements show negative magnetic helicity in northern hemisphere and positive in the southern (Seehafer 1990; Pevtsov et al. 1995, 2001; Abramenko et al. 1997; Bao & Zhang 1998).

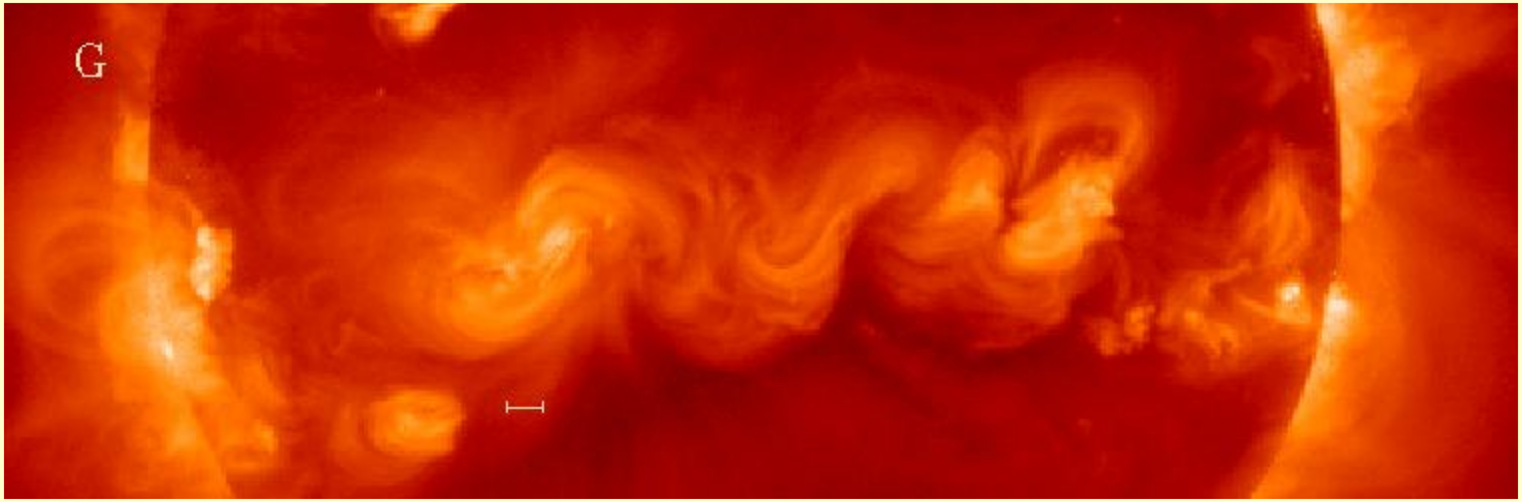
A quantitative measure of helicity:

$$\alpha = \frac{(\nabla \times \boldsymbol{B})_z}{B_z}$$



From Pevtsov, Canfield & Metcalf 1995

Coronal loop seen by Yohkoh



Martin et al. 1992, 1993 – Coronal filaments also have opposite polarities in the two hemispheres

Beiber et al. 1987; Smith & Bieber 1993 – Interplanetary magnetic field also has opposite polarities above & below equatorial plane

What gives rise to magnetic helicity?

Magnetic field is generated by dynamo process

Flux tubes rise due to magnetic buoyancy through convection zone and produce active regions

Longcope, Fisher & Pevtsov (1998) – Helical turbulence in covection zone imparts helicity to rising flux tubes (independent of solar cycle)

Choudhuri (2003); Choudhuri, Chatterjee & Nandy (2004), Chatterjee, Choudhuri & Petrovay (2006) – Helicity generation is linked to the dynamo process

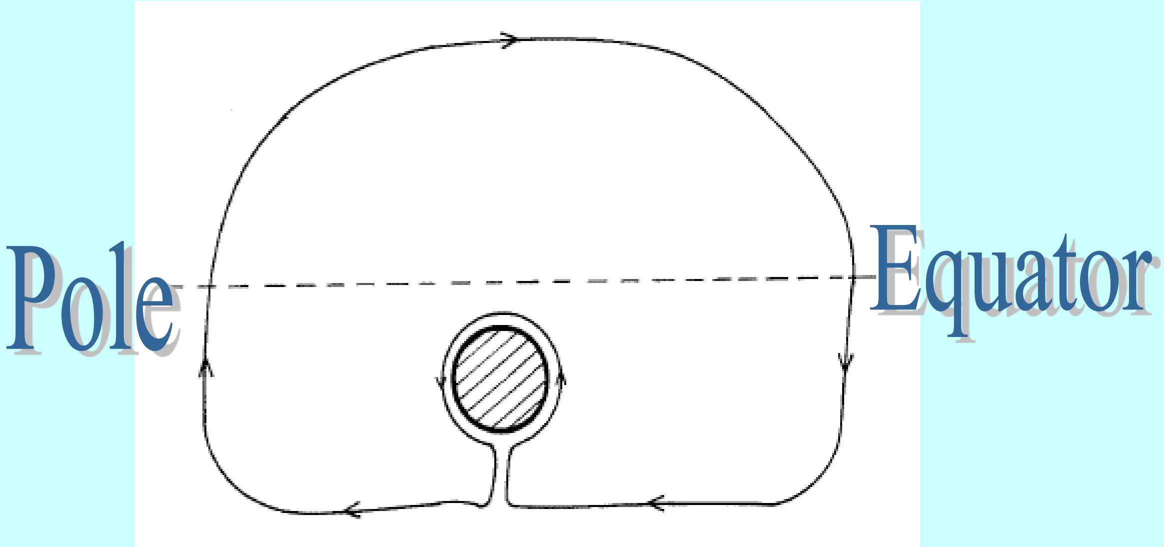
α-effect produces magnetic helicity of the **same sign** as α

Babcock-Leighton process => α-coefficient concentrated near the solar surface (positive in northern hemisphere) => Will it produce positive helicity in northern hemisphere?

Dynamo equation deals with **mean fields**, but helicity is associated with **flux tubes**!

Choudhuri (2003) studied the connection between mean field theory and flux tubes

From Choudhuri (2003)



Northern hemisphere

B inside flux tube into the slide

Poloidal field accreted around flux tube gives negative helicity

It is difficult to change magnetic helicity (Woltjer 1958; Taylor 1974; Berger 1985)

Dynamo process generates helicity of opposite sign in small and large (mean field) scales – Seehafer (1990)

Flux tube to be associated with small scales

At mean field scales, averaging over flux tubes gives positive helicity

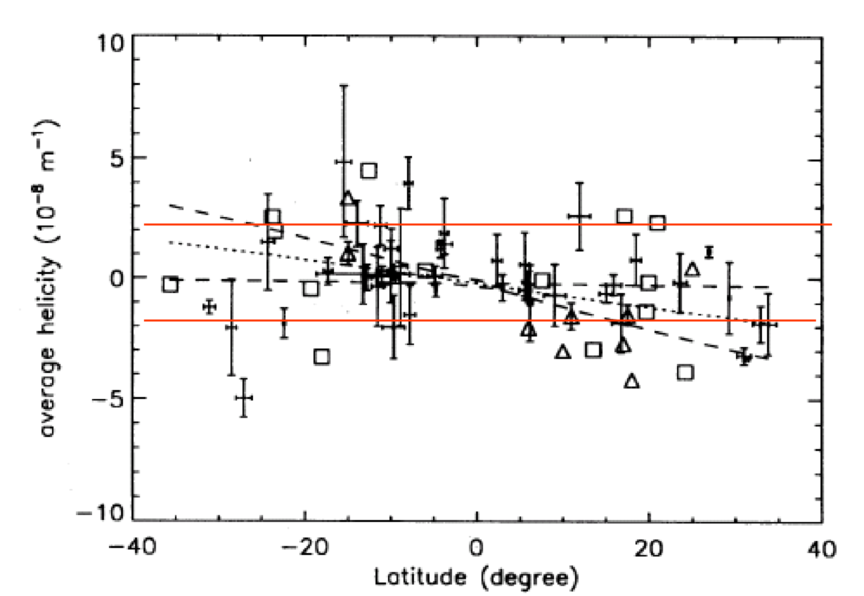
Estimate of magnetic helicity (Choudhuri, Chatterjee & Nandy 2004)

$$\Phi = B_p d_{\rm scz}$$

$$J \sim \frac{\Phi}{r_{ft}^2}$$

$$\alpha \sim \frac{J}{B_T} \Rightarrow \alpha \sim \frac{B_p d_{\rm scz}}{B_T r_{ft}^2}$$

Assuming $B_p \sim 1$G and $B_T \sim 3000$G,
$r_{ft} \sim 2000$ km $\rightarrow \alpha \sim 2 \times 10^{-8}$ m^{-1}.



Very simple estimate gives the correct order of magnitude !!!

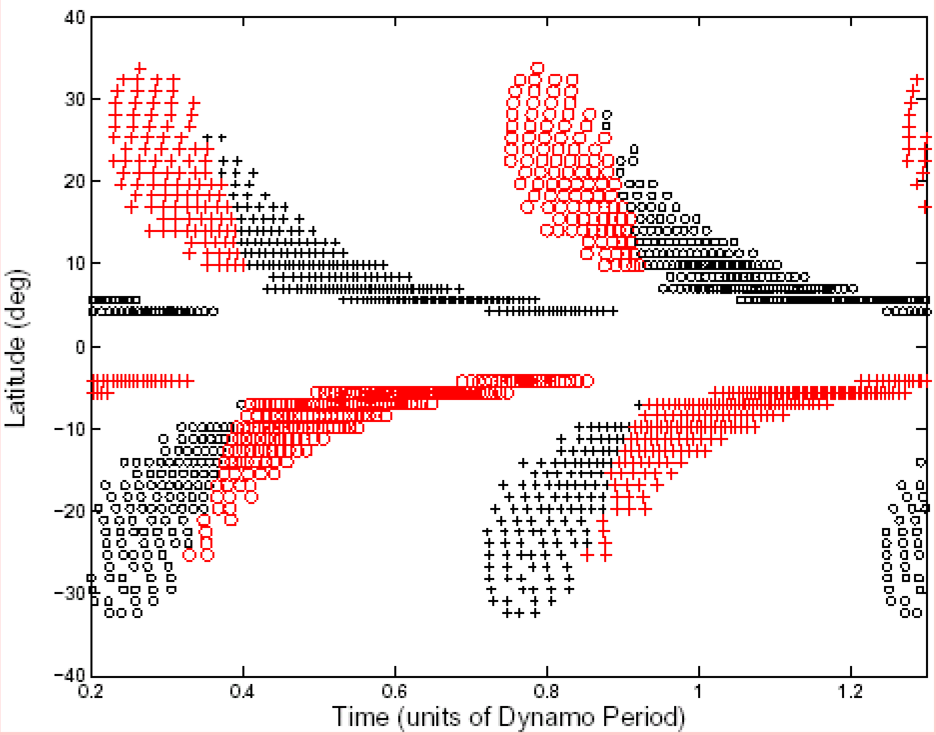
Helicity calculation from our dynamo model
(Choudhuri, Chatterjee & Nandy 2004)

Flux eruption whenever
$B > B_C$ above $r = 0.71R$

⇒ Calculate helicity

Red : positive helicity

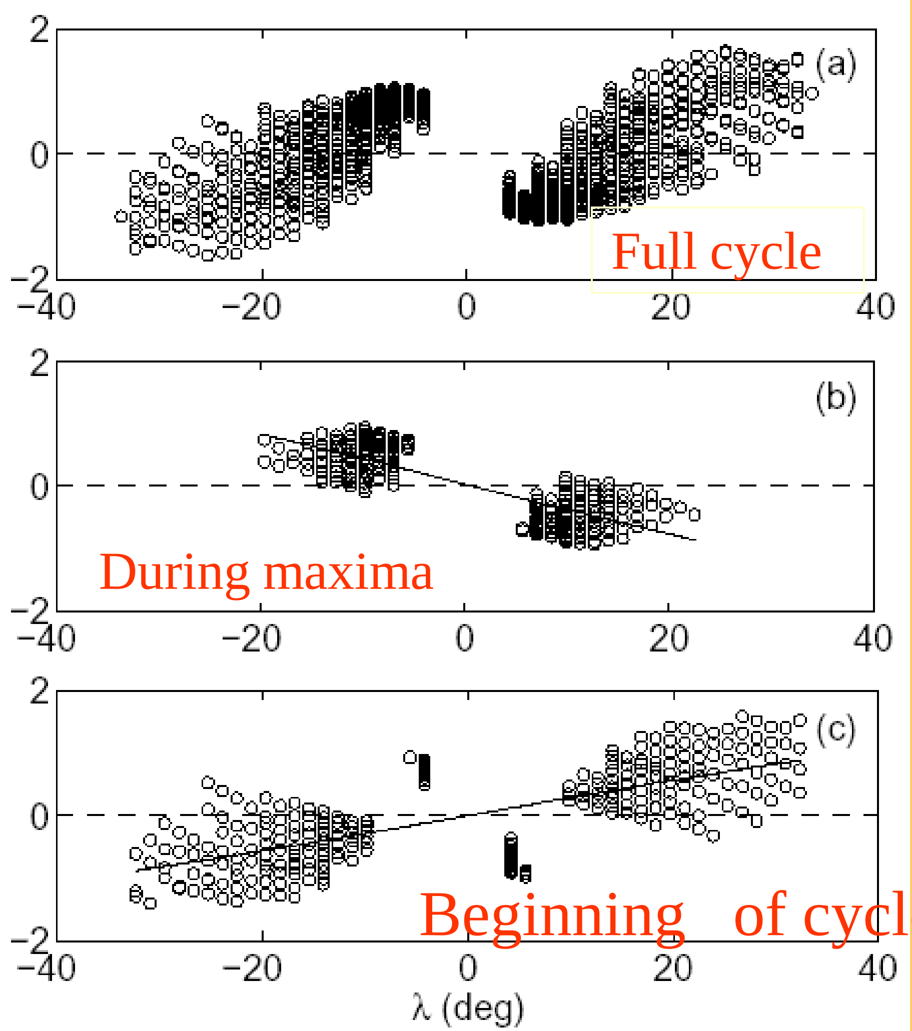
Black: negative helicity

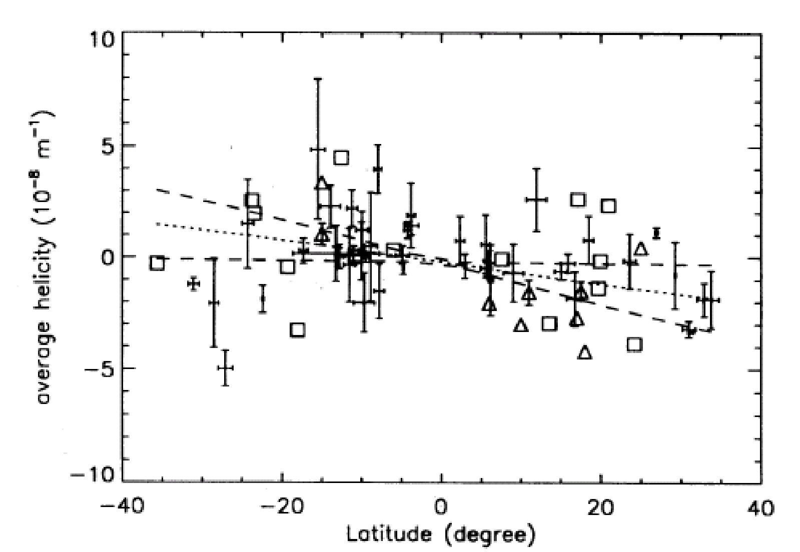


Correct helicity during sunspot maxima (negative in north & positive in south)

Helicity reversal at the beginning of a cycle!

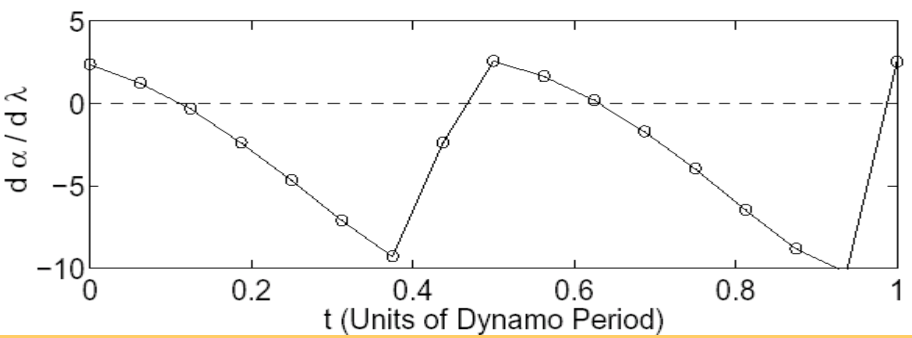
Helicity at different latitudes – from Choudhuri, Chatterjee & Nandy (2004)





Not too bad match with observations!

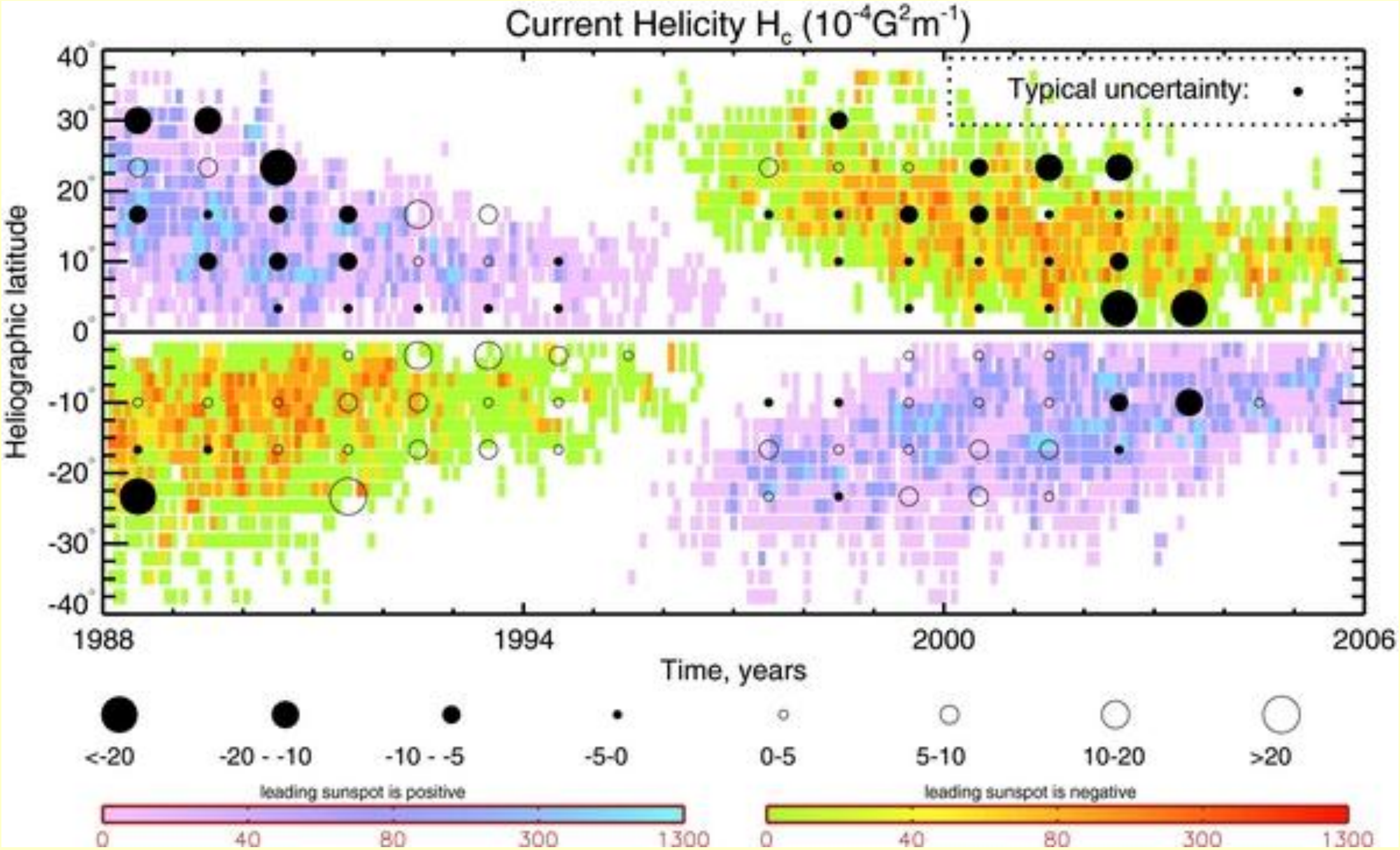
Fluctuations to be included



Cycle variation from our model

Cycle variation reported by Bao et al. (2000); Hagino & Sakurai (2005)

Butterfly diagram for helicity



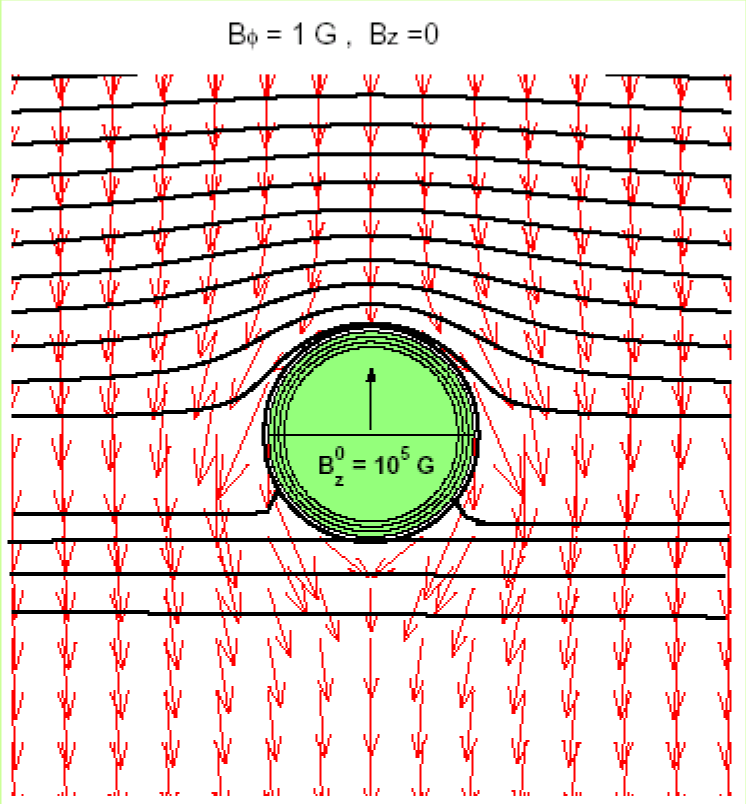
From Zhang et al. (2012)

Observational studies of possible cycle variations of helicity: Bao & Zhang 1998; Hagino & Sakurai 2004; Tiwari et al. 2009; Zhang et al. 2010; Hao & Zhang 2011; Zhang et al. 2012

Build-up of helicity during the rise of the flux tube through convection zone (Chatterjee, Choudhuri & Petrovay 2006)

Accreted poloidal flux penetrates inside the flux tube due to turbulent diffusion (suppressed inside flux tube)

Sunspot decay by nonlinear diffusion studied by Petrovay & Moreno-Insertis (1997)



1-D model with radially inward flow!

2-D calculations under progress

Magnetic field evolution equations in Lagrangian coordinates:

$$\frac{\partial B'_z}{\partial t} = F^2 \frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\eta \xi \frac{\partial B'_z}{\partial \xi} \right),$$

$$\frac{\partial B'_\phi}{\partial t} = F^2 \frac{\partial}{\partial \xi} \left[\eta \frac{1}{\xi} \frac{\partial}{\partial \xi} (\xi B'_\phi) \right] - F \frac{\partial}{\partial \xi} (\mathbf{v} B'_\phi),$$

where $F = \sqrt{R_t \rho_e / R_b \rho_{e,0}}$,

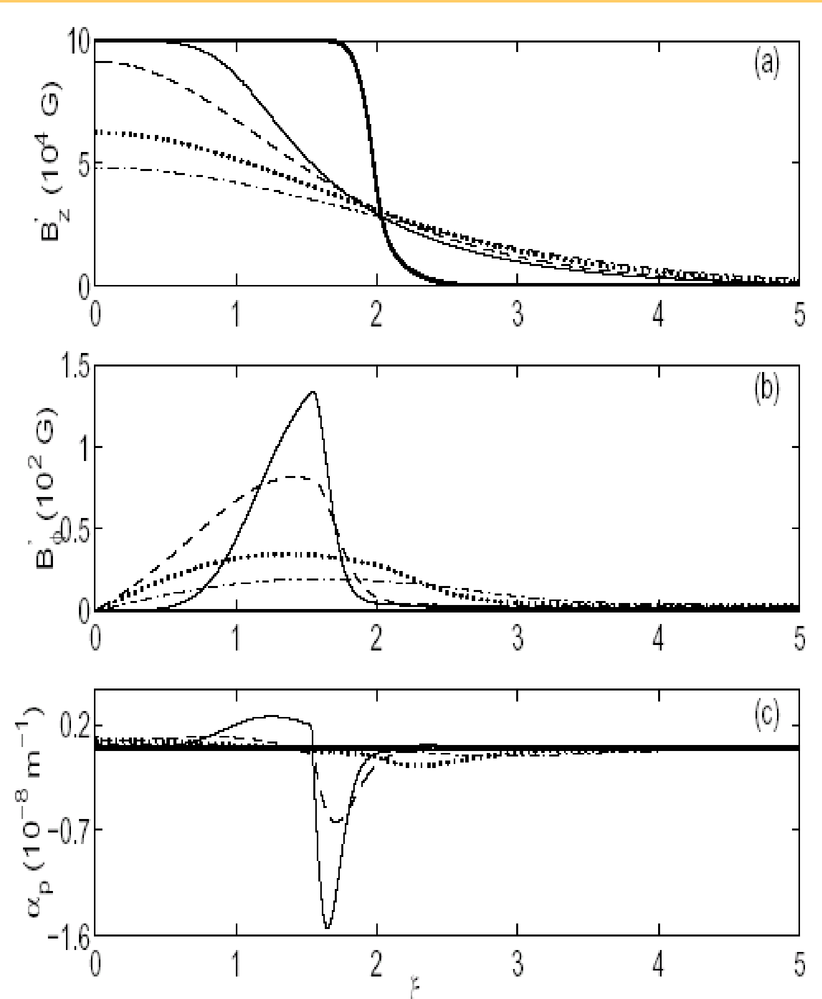
Turbulent diffusivity with magnetic quenching and Kolmogorov scaling (following Petrovay & Moreno-Insertis 1997):

$$\eta = \frac{\eta_{00} (r_{ft}/H)^{4/3}}{1 + |B/B_{eq}|^\kappa},$$

with $\eta_{00} = 3 \times 10^{12}\ \mathrm{cm^2 s^{-1}},\ \kappa = 2$

Results from Chatterjee, Choudhuri & Petrovay (2006)

Magnetic Field falling to low values



$0.7\ R_\odot$ Thick solid

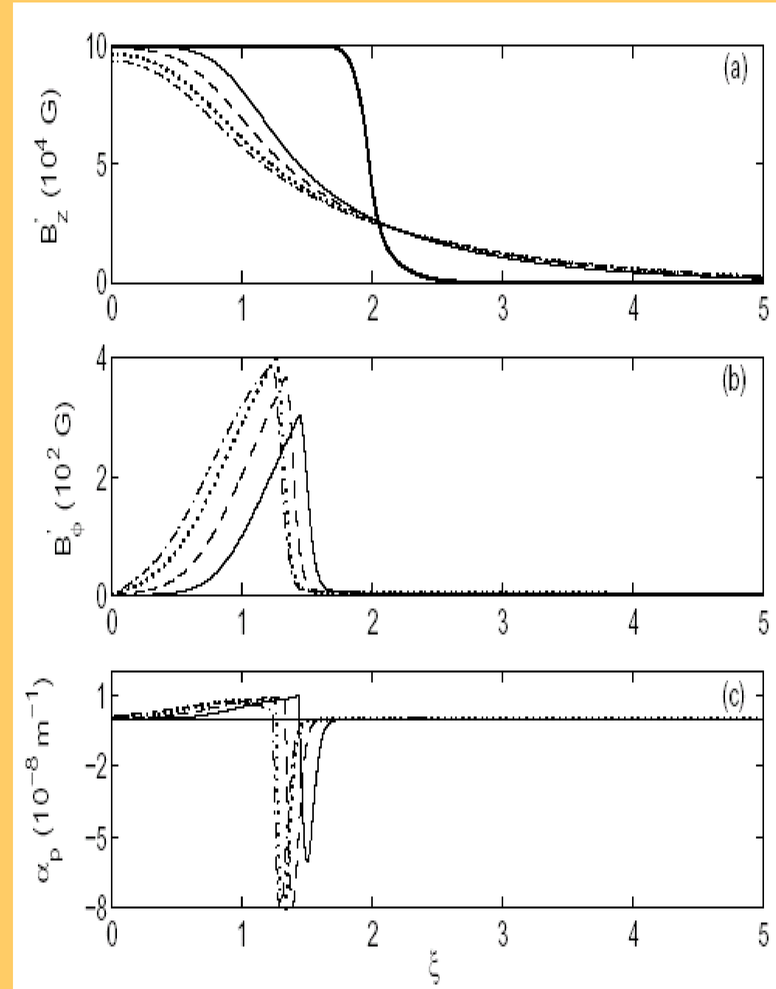
$0.85 R_\odot$ Solid

$0.9 R_\odot$ Dashed

$0.95 R_\odot$ Dotted

$0.98 R_\odot$ Dash-dotted

Mag. Field restricted above $3B_{eq}$



More penetration into flux tube if magnetic field becomes weak in top layers

Detailed comparisons between observation & theory may be possible in future!

Conclusions

- The flux transport dynamo explains many aspects of the sunspot cycle, though there are uncertainties about values of some parameters
- Magnetic buoyancy is a 3D process and cannot be included fully satisfactorily in a 2D model
- To produce 10^5 G magnetic fields inside flux tubes at the base of convection zone, you need to start from polar field concentrations of order a few hundred G
- Magnetic helicity can be produced by poloidal field getting wrapped around rising flux tubes
- This model predicts a reversal of hemispheric helicyity sign rule at the beginning of a sunspot cycle