From kinematics to dynamics: Meridional circulation and torsional oscillations

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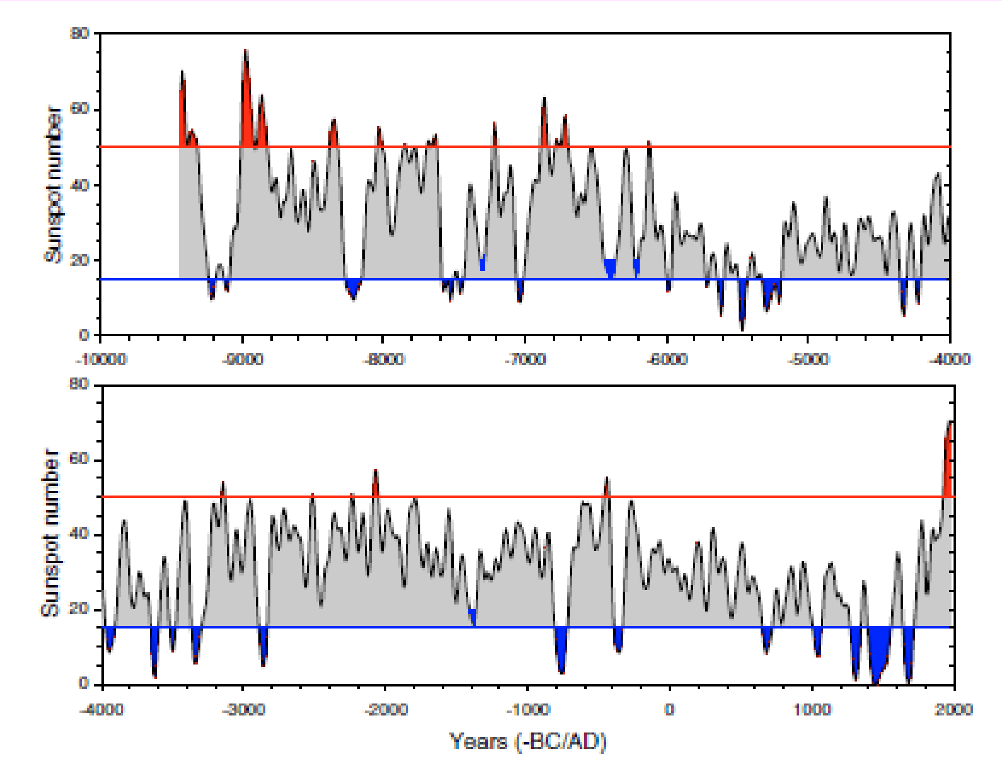
Spill-over from Lecture 3
First ~15 minutes

Theoretical modelling of grand minima from the flux transport dynamo model

History of solar activity before telescopic records reconstructed by Eddy (1977), Stuiver & Braziunas (1989),Voss et al. (1996), Usoskin, Solanki & Kovaltsov (2007)



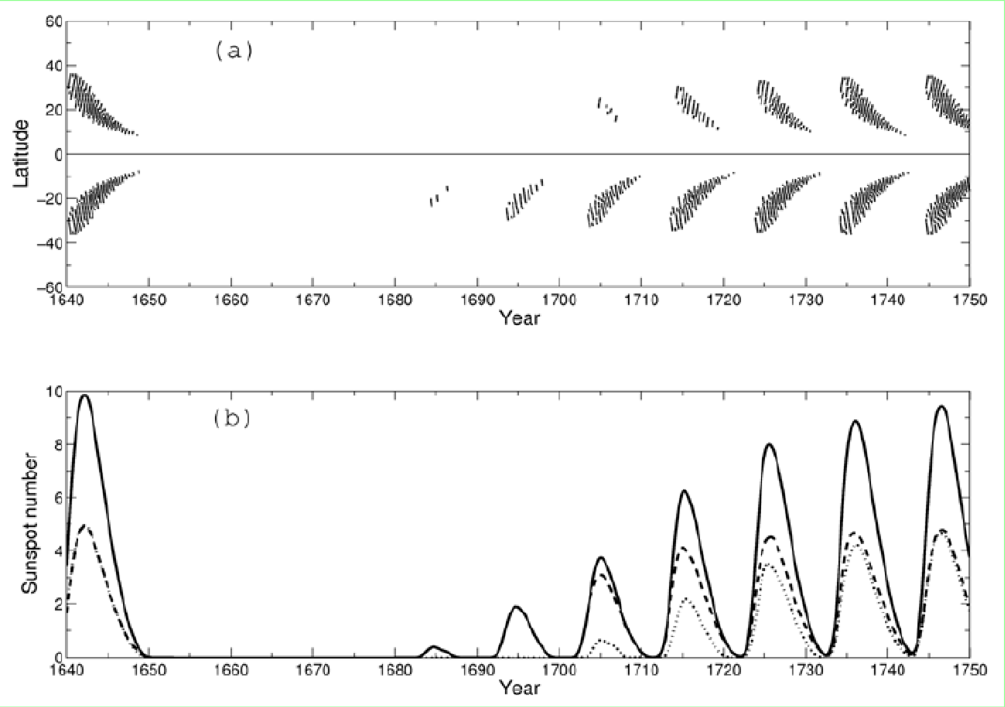
From Usoskin, Solanki & Kovaltsov (2007) – 27 grand minima and 19 grand maxima in the last 11,000 years!

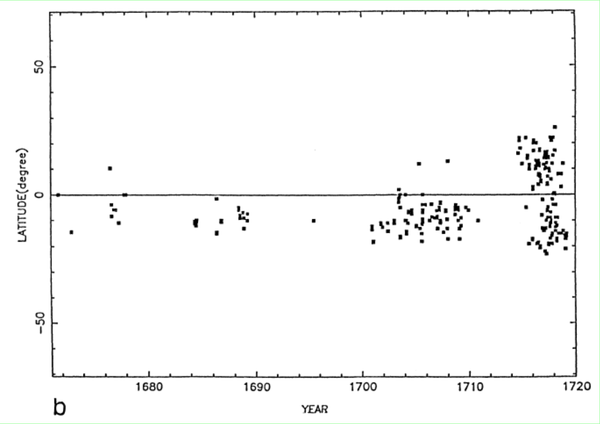
Possible mechanisms for producing grand minima

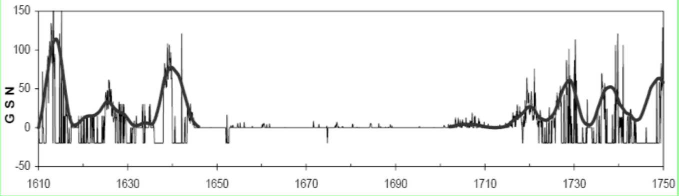
- Fluctuations in Babcock-Leighton process may make the poloidal field weak
- Fluctuations in meridional circulation may make it very weak

Choudhuri & Karak (2009) - Modelling of Maunder minimum with flux transport dynamo

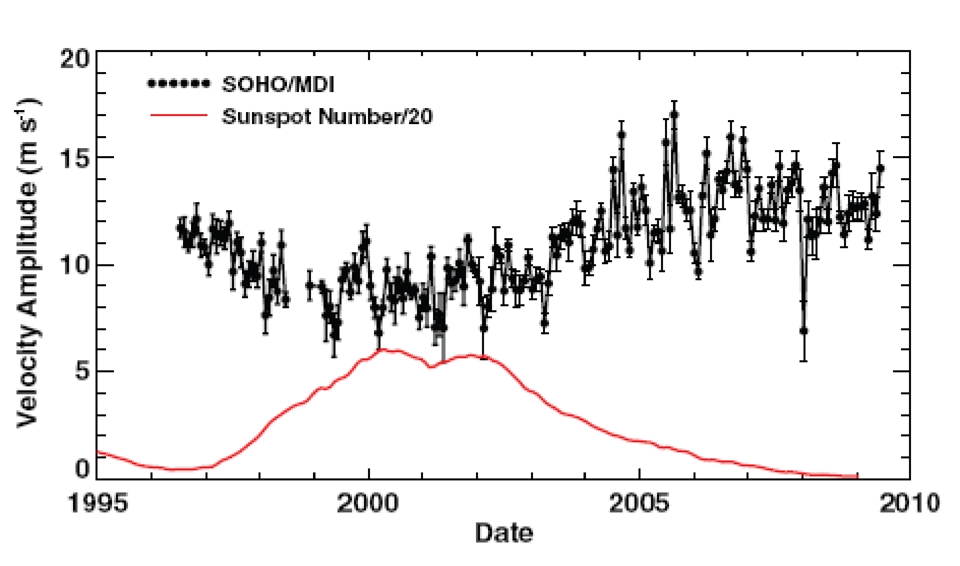
Assumption : Poloidal field drops to 0.0 and 0.4 of its average value in the two hemispheres







Period of flux transport dynamo ~ inverse of meridional circulation speed



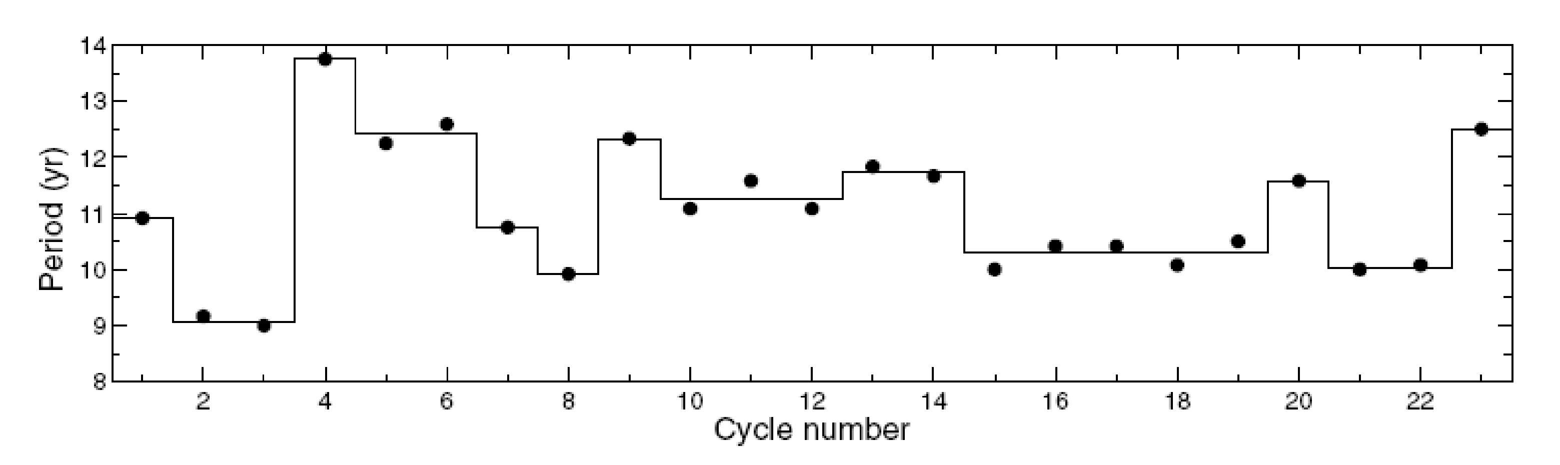
Decreases at sunspot maxima (Hathaway & Rightmire 2010)

Due to Lorentz force?

Does not cause irregularities (Karak & Choudhuri 2011)

We disagree with Nandy, Munoz-Jaramillo & Martens (2011)

Possible long-term fluctuations in meridional circulation from inverse of cycle durations (Karak & Choudhuri 2011)



Suppose meridional circulation slows down

Dynamo period increases

(Yeates, Nandy & Mackay 2008)

Diffusion has more time to act on the fields

Cycles become weaker

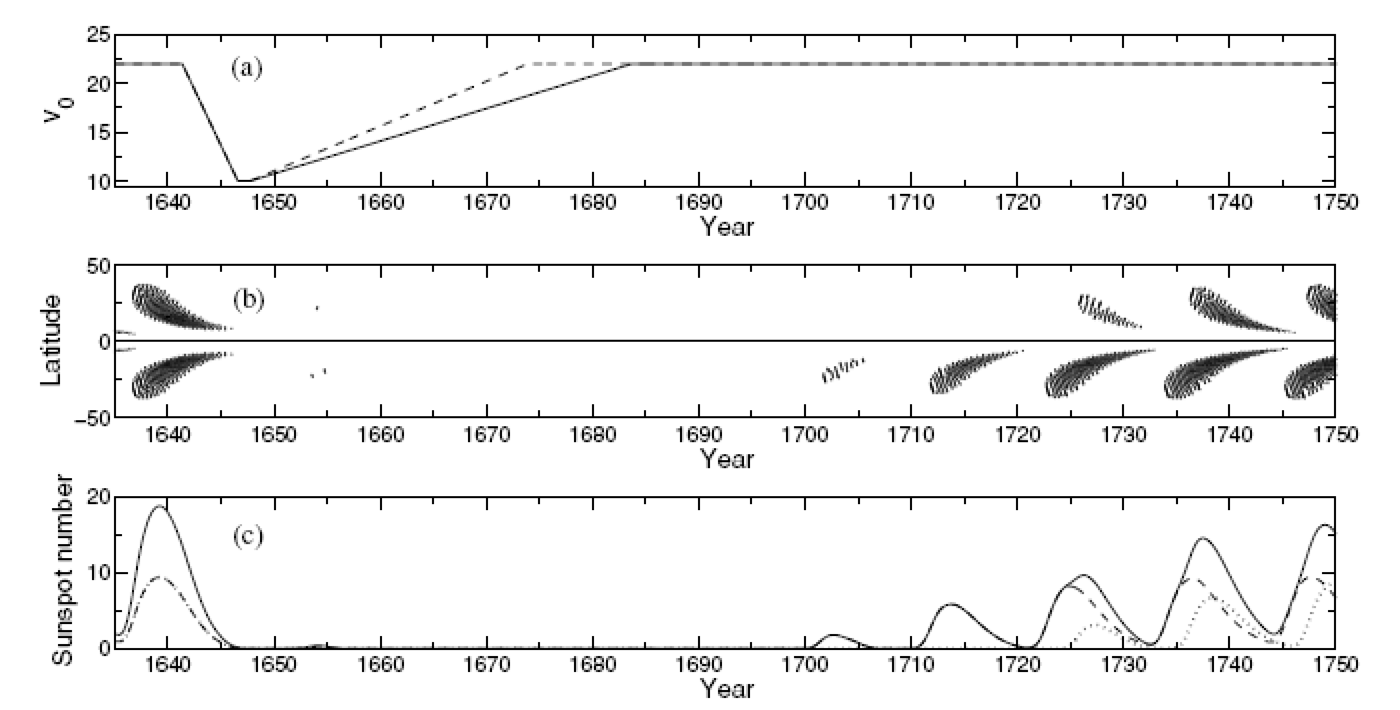
Applicable for high diffusivity dynamo

Differential rotation generates more toroidal flux

Cycles becomes stronger

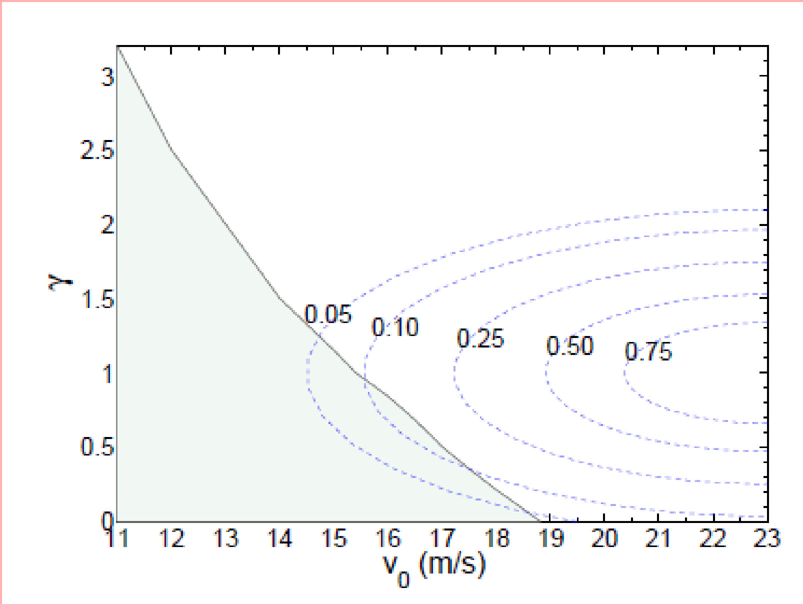
Applicable for low diffusivity dynamo

Karak (2010) found that a sufficiently large decrease in meridional circulation can cause grand minimum



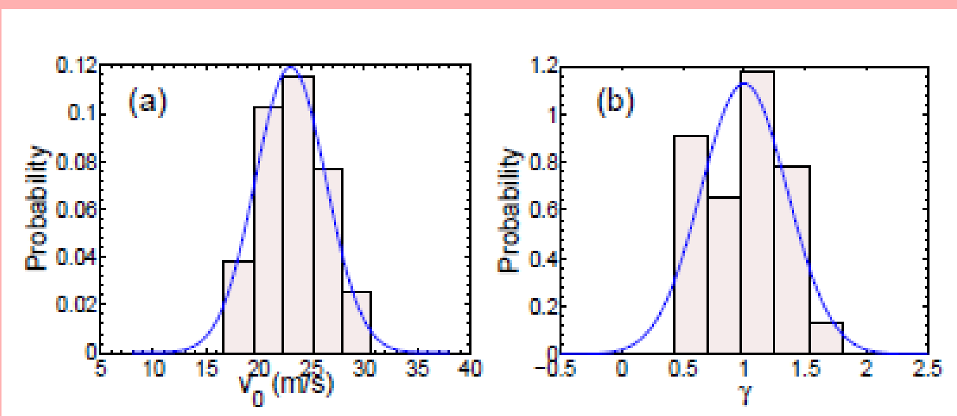
Periods during grand minimum should be longer!

From Choudhuri & Karak (2012)



If poloidal field γ and meridional circulation v_0 at the beginning of a cycle lie in the shaded region, then we get grand minimum

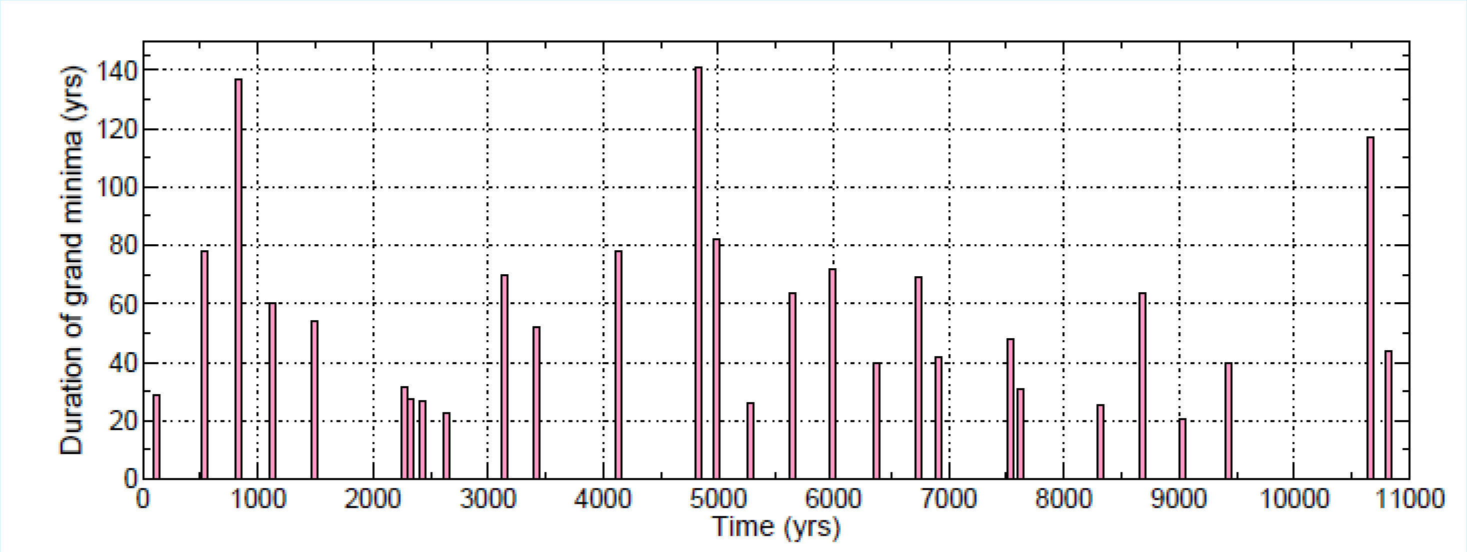
Durations of last 28 cycles – meridional circulation fluctuations
Strengths of last 28 cycles – polar field fluctuations



$$P(\gamma, v_0)d\gamma dv_0 = \frac{1}{\sigma_v\sqrt{2\pi}}\exp\left[-\frac{(v_0-\overline{v_0})^2}{2\sigma_v^2}\right]$$

$$\times\frac{1}{\sigma_\gamma\sqrt{2\pi}}\exp\left[-\frac{(\gamma-1)^2}{2\sigma_\gamma^2}\right]d\gamma\, dv_0$$

$\int P(\gamma, v_0)\, d\gamma\, dv_0$ integrated over the shaded area gives 1.3% probability of a cycle getting into grand minima (Choudhuri & Karak 2012)



One of the runs of our dynamo code with fluctuations

We get 20 -30 grand minima in 11,000 years!!!

Recovery mechanism from grand minima poorly understood

Our theoretical understanding of velocity patterns in the convection zone (differential rotation, meridional circulation) is very limited.

Until we can do successful DNS of velocity fields, we have no hope for realistic DNS of the dynamo.

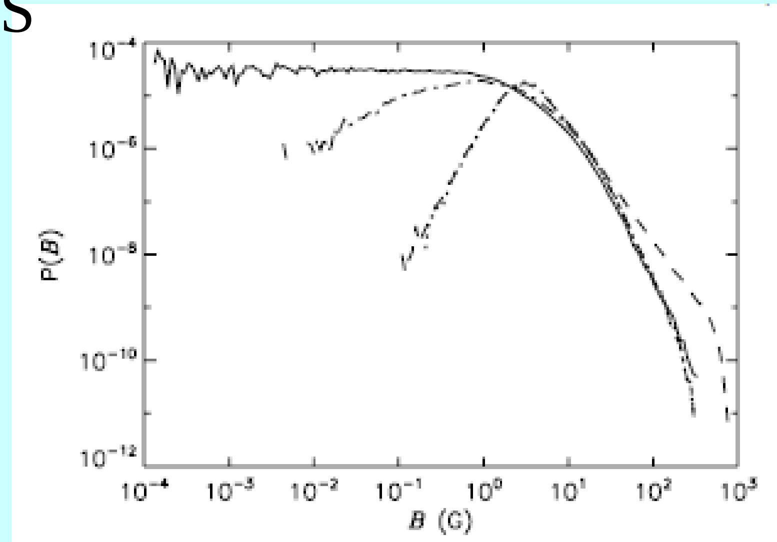
Recent DNS are of exploratory nature and have not reached the stage of detailed comparison with observations (Brandenburg, Nordlund, Brun, Miesch, Toomre)

Kinematic models use the velocity fields discovered by helioseismology and are able to model many aspects of observational data by assigning suitable values to different parameters

Small scale dynamo - MHD turbulence stretches out seed magnetic fields until they become strong enough to resist further stretching

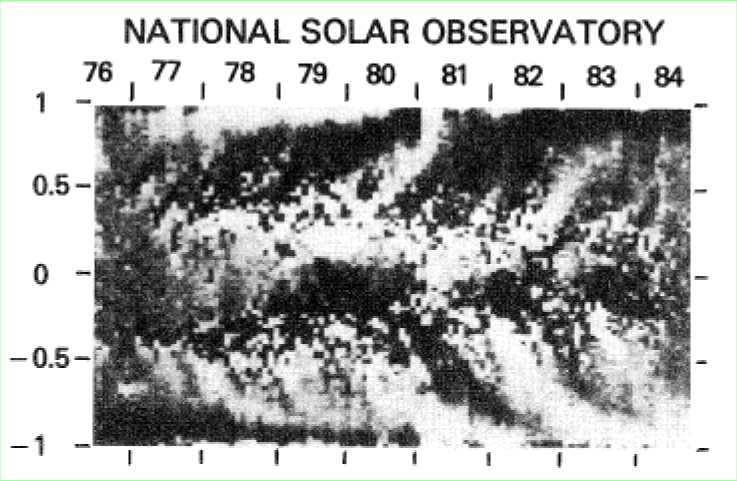
DNS carried out by many groups (Cattaneo 1999; Graham, Danilovic & Schussler 2009)

Comparison between Hinode data and small scale dynamo DNS



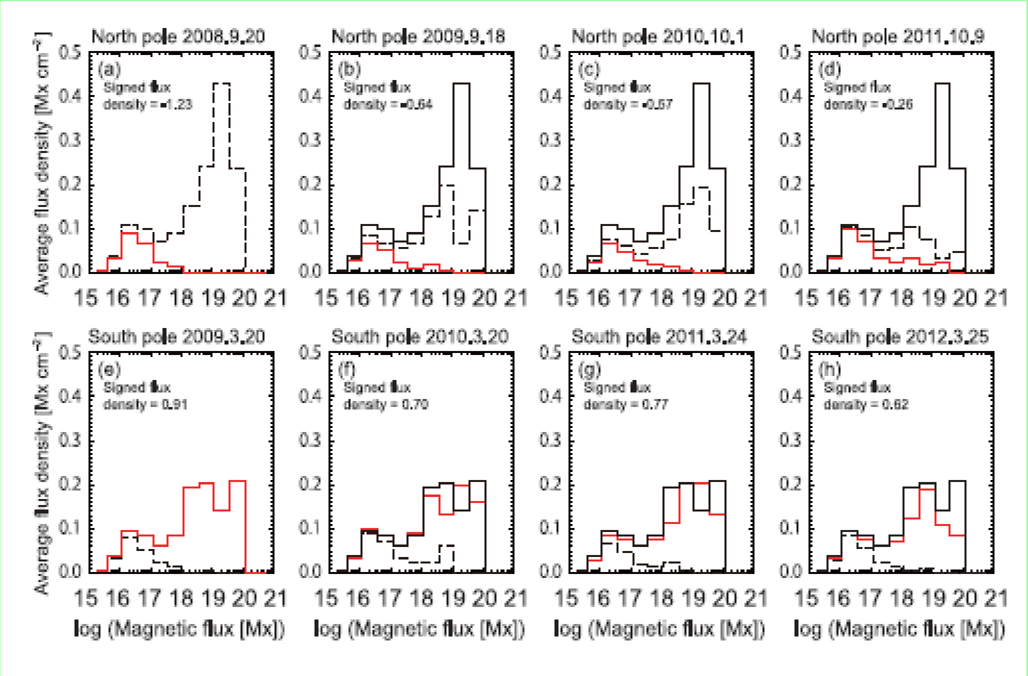
From Graham, Danilovic & Schussler (2009)

Any possible relation between global dynamo and small scale dynamo?



Time-latitude plot of magnetic field outside active regions => clear contributions from the global dynamo

From Shiota et al. (2012)
Small flux concentrations due to small scale dynamo and large ones due to global dynamo?



Dynamics of differential rotation and meridional circulation

Navier-Stokes equation should be be starting point
Thermodynamics is often important
Turbulence in convection zone has to be handled – mean field theory?

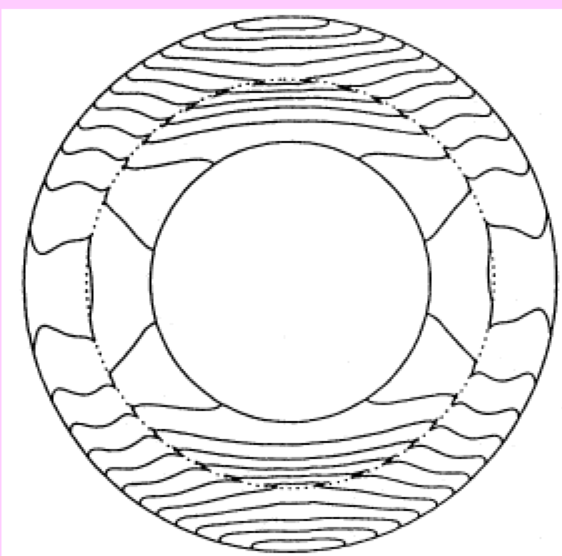
Classic study by Kippenhahn (1963)
Isotropic viscosity => Solid body rotation
Radial viscosity bigger => equatorial deceleration, poleward circulation at surface
Radial viscosity smaller => equatorial acceleration, equatorward circulation at surface

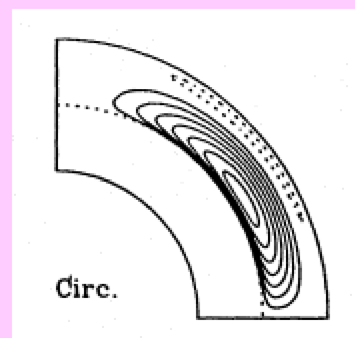
Kitchatinov & Rudiger (1995) calculated the turbulent stresses in the convection zone from their turbulence model

$$Q_{ij} = Q^{\Lambda}_{ij} - \mathcal{N}_{ijkl} \frac{\partial \bar{u}_k}{\partial x_l},$$

$$\begin{aligned}
\mathcal{N}_{ijkl} = {} & \nu_1 \left(\delta_{ik}\delta_{jl} + \delta_{jk}\delta_{il} \right) \\
& + \nu_2 \left(\delta_{il} \frac{\Omega_j \Omega_k}{\Omega^2} + \delta_{jl} \frac{\Omega_i \Omega_k}{\Omega^2} + \delta_{ik} \frac{\Omega_j \Omega_l}{\Omega^2} + \right. \\
& \left. + \delta_{jk} \frac{\Omega_i \Omega_l}{\Omega^2} + \delta_{kl} \frac{\Omega_i \Omega_j}{\Omega^2} \right) + \nu_3 \delta_{ij}\delta_{kl} - \nu_4 \delta_{ij} \frac{\Omega_k \Omega_l}{\Omega^2} \\
& + \nu_5 \frac{\Omega_i \Omega_j \Omega_k \Omega_l}{\Omega^4}
\end{aligned}$$

Their results on differential rotation and meridional circulation





~5^{O} pole-equator temperature difference needed to drive the meridional circulation

Lorentz force varies periodically with the solar cycle

Does it produce any observable motion?

Kinds of motion expected =>

Look at the Navier-Stokes equation

φ-component =>

Periodic variation of rotation velocity (torsional oscillations)

(r, θ)-component =>

Periodic variation of meridional circulation

Model of torsional oscillations

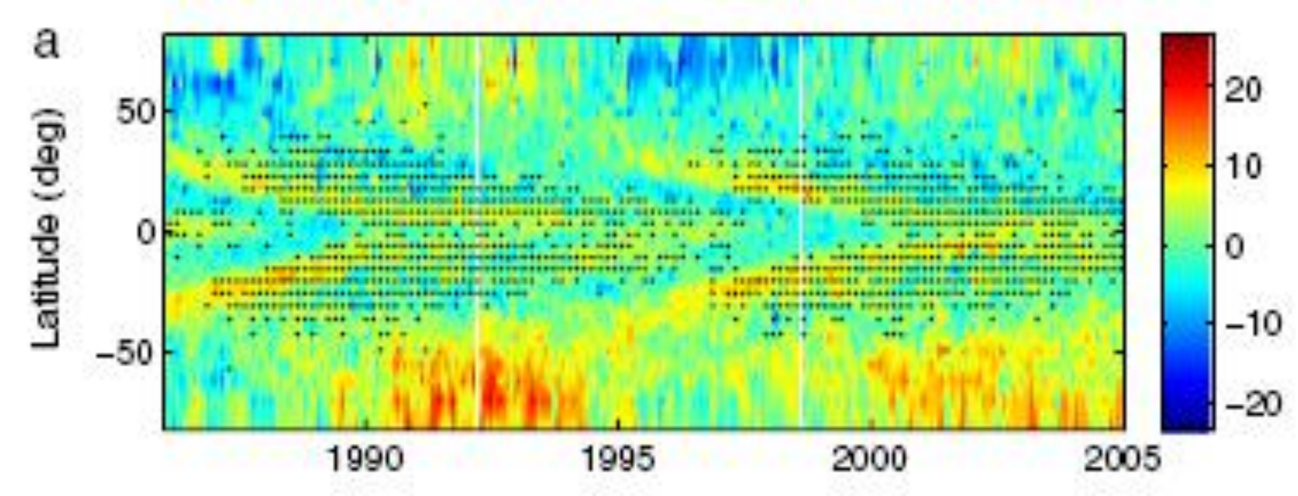
(Chakraborty, Choudhuri & Chatterjee 2009)

Discovered at the surface by Howard & LaBonte (1980)

Helioseismic observations of oscillations within the convection zone (Kosovichev & Schou 1997; Vorontsov et al. 2002; Basu & Antia 2003; Howe et al. 2005)

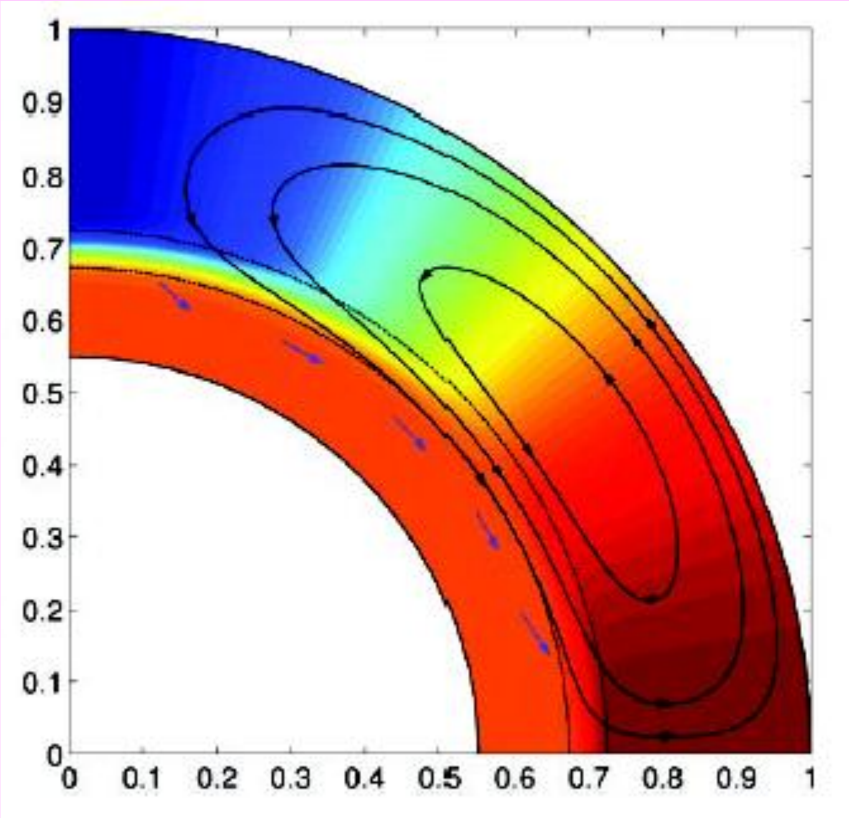
Previous theoretical models (Durney 2000; Covas et al. 2000; Bushby 2006; Rempel 2006) cannot explain the early initiation at higher latitudes.

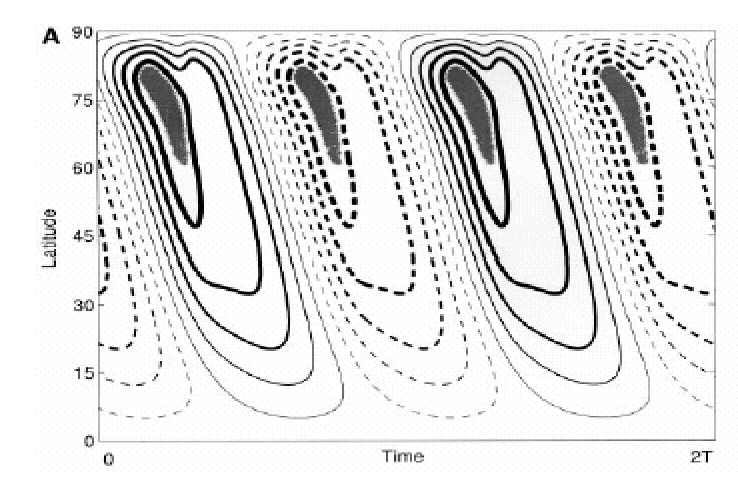
Salient observational features



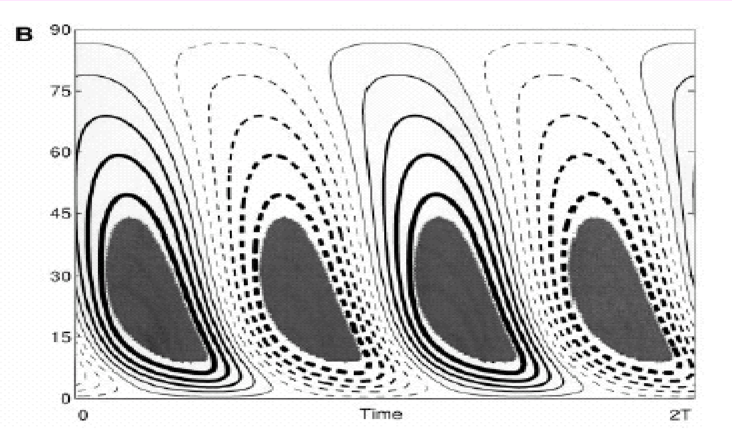
- *The amplitude of oscillation is ~ 5 m s^{-1} near the surface ~ 1% of Ω.*
- *Two branches: Poleward propagating, and equatorward propagating, extending throughout convection zone.*
- *Equatorward propagating branch begins 2-3 years before the first sunspot eruptions of a cycle at a higher latitude. Apparent violation of causality*

Nandy & Choudhuri (2002) introduced meridional flow penetrating slightly below the tachocline





Without penetrating flow



With penetrating flow

Strong toroidal fields build up at high latitudes a few years before sunspot eruptions of the cycle begin

Theoretical Model

Along with the equations of the flux transport dynamo, we solve the Navier-Stokes equation for v_φ

$$\rho\left[\frac{\partial v_\phi}{\partial t} + \frac{v_r}{r}\frac{\partial}{\partial r}(r v_\phi) + \frac{v_\theta}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta v_\phi)\right] = (\mathbf{F}_L)_\phi$$

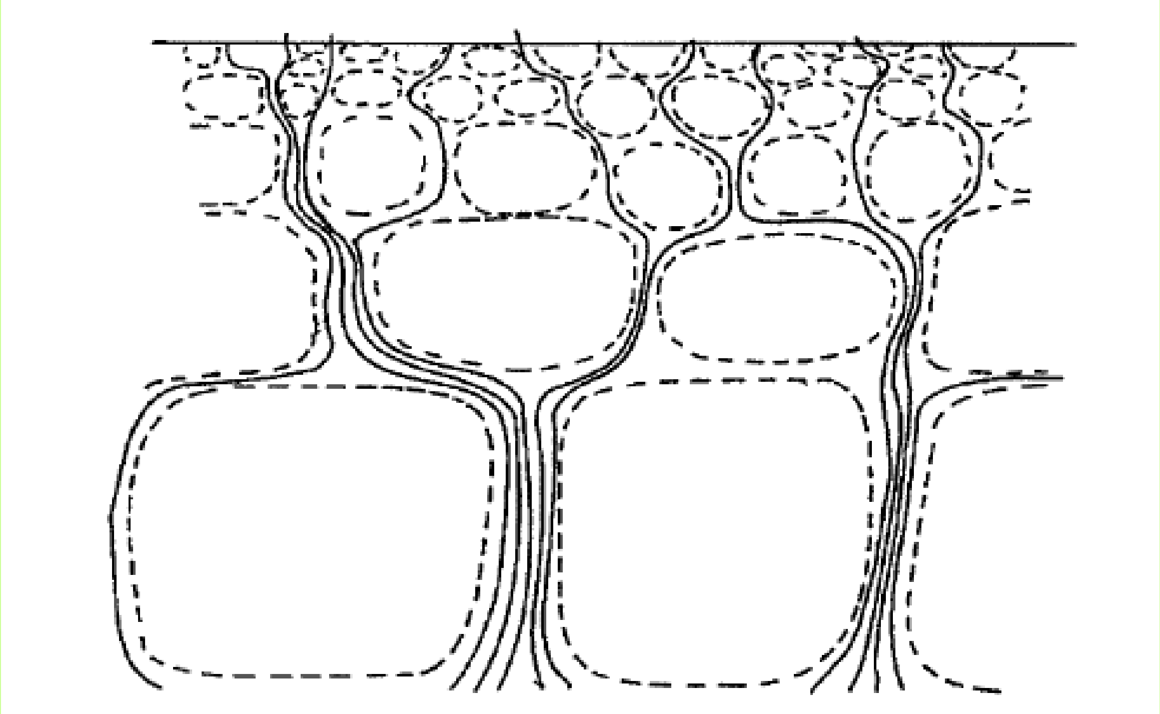
$$+\frac{1}{r^3}\frac{\partial}{\partial r}\left[\nu\rho r^4\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left[\nu\rho\sin^3\theta\frac{\partial}{\partial\theta}\left(\frac{v_\phi}{\sin\theta}\right)\right],$$

where

$$4\pi(\mathbf{F}_L)_\phi = \frac{1}{s^3}J\left(\frac{sB_\phi, sA}{r, \theta}\right),$$

is the Lorentz force of the magnetic field

$$\mathbf{B} = B(r, \theta, t)\mathbf{e}_\phi + \nabla \times [A(r, \theta, t)\mathbf{e}_\phi],$$



Choudhuri (2003) argued on the basis of some dynamo requirements that magnetic field in the convection zone should be like this

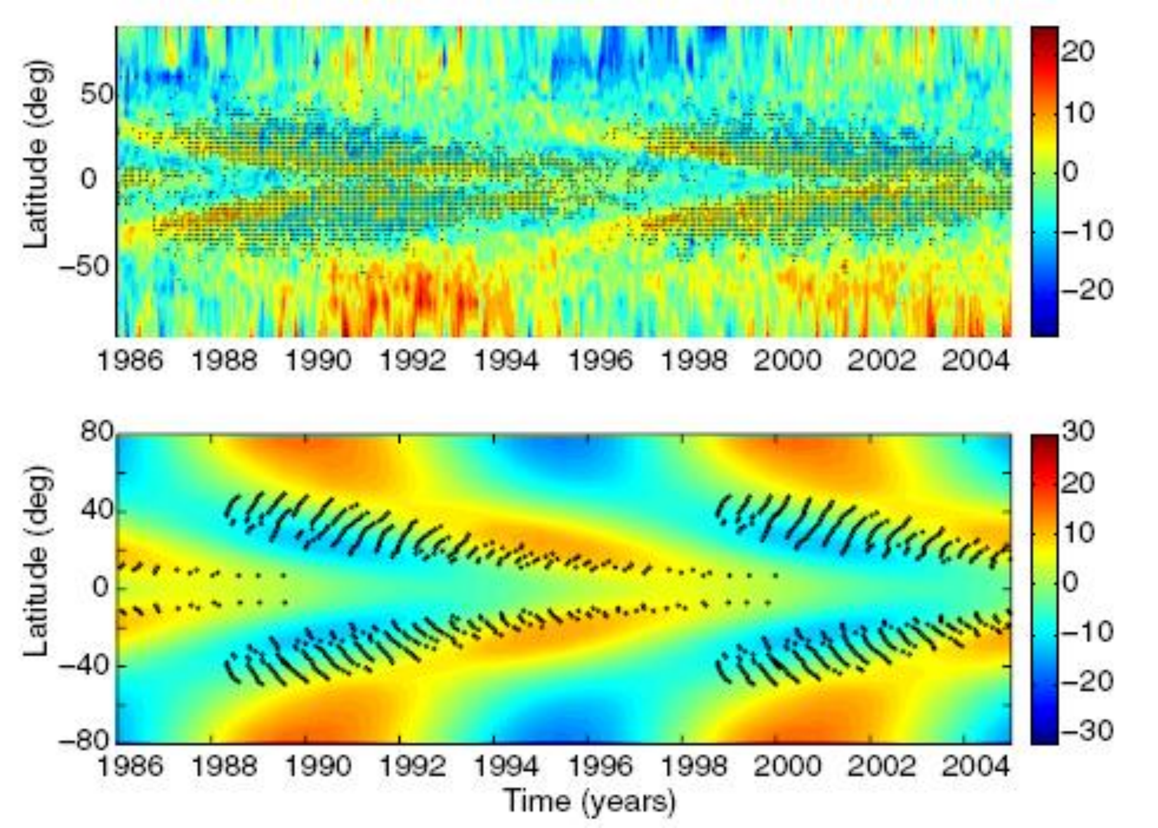
First prediction of strong flux concentrations in the polar regions – later discovered by Hinode (Tsuneta et al. 2008)

Magnetic stresses built up at the based of the convection zone can be transported upward by Alfven waves (travel time ~ 2 – 3 years)

Comparison between theory and observations

Torsional oscillations at the surface

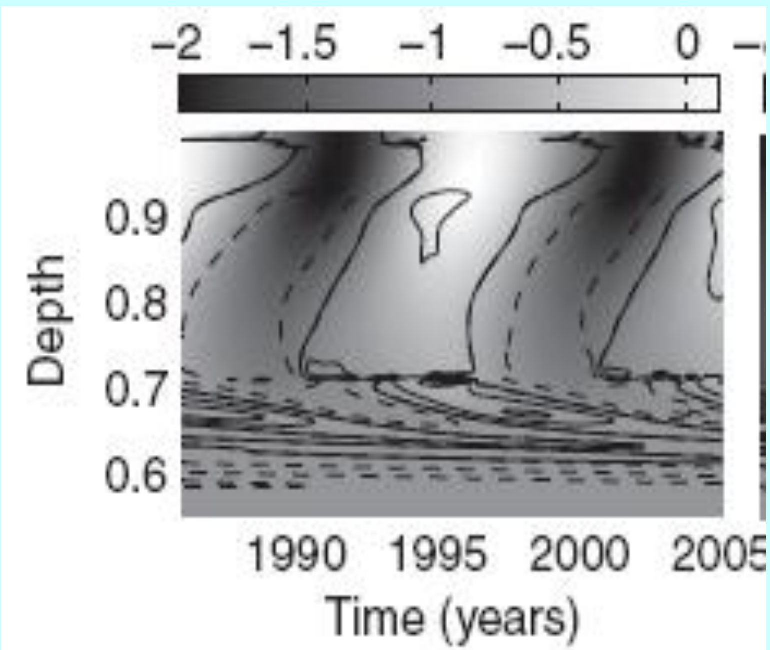
From Chakraborty, Choudhuri & Chatterjee (2009)



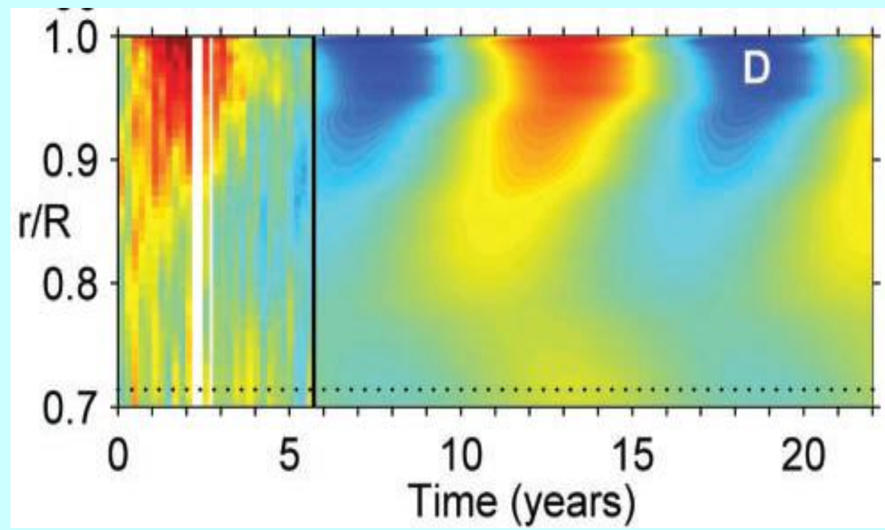
The surface data is from Mount Wilson (Courtesy: R. Ulrich)

Comparison between theory and observations

Depth-time plots at latitude of 20º



From our paper

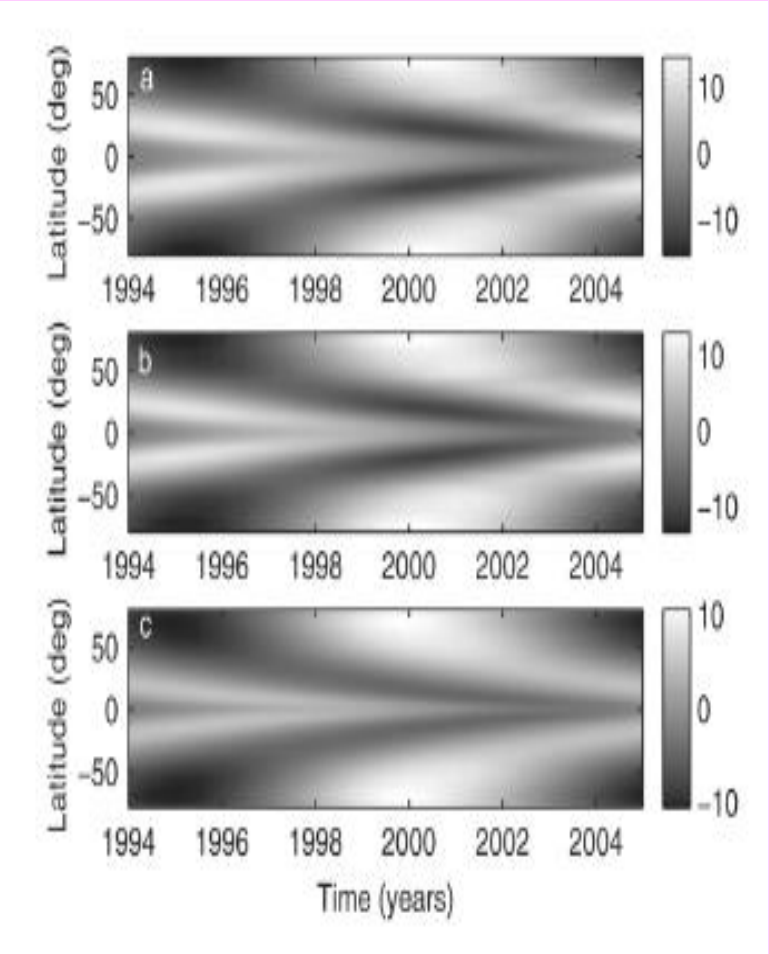


From Vorontsov et al. (2002)

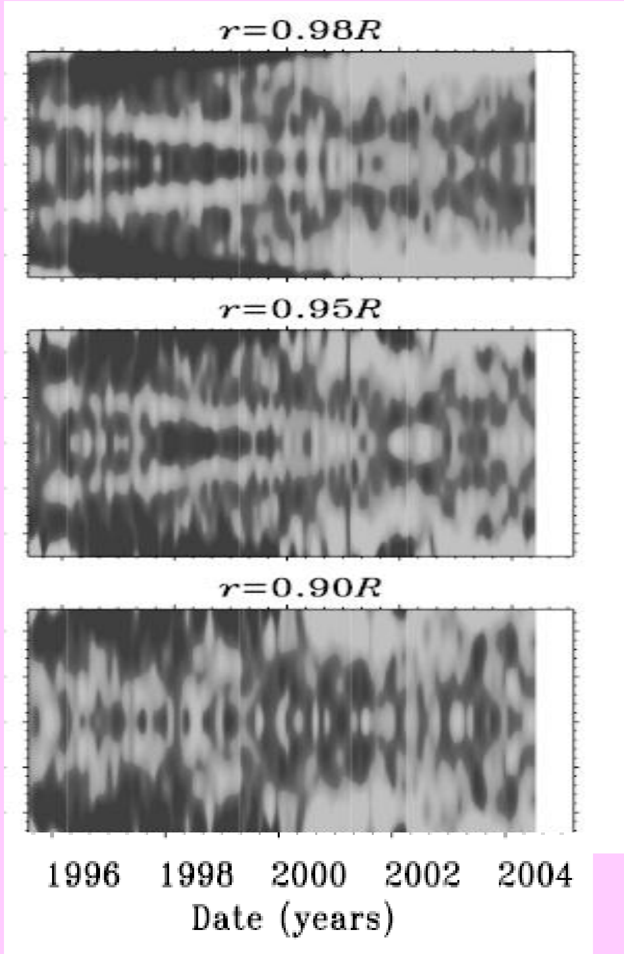
Based on 1996 – 2002 data

Comparison between theory & observations

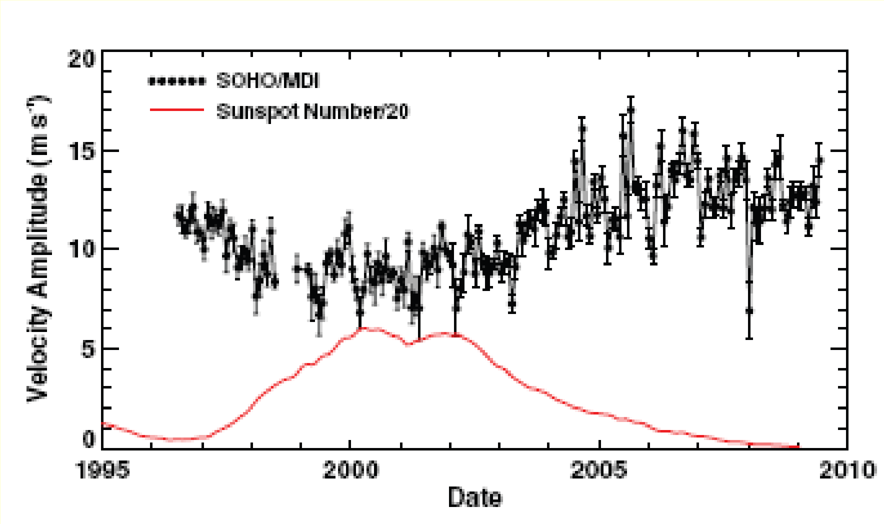
Time-latitudes plots at different depths



From our paper (0.95R, 0.9R, 0.8R)



From Howe et al. (2005)

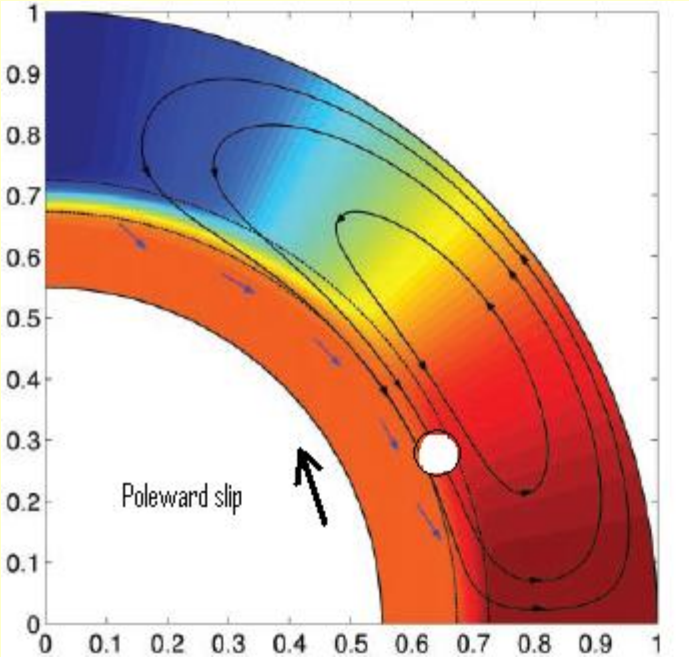


From Hathaway & Rightmire (2010) – Variation of meridional circulation with solar cycle

Toroidal field at bottom of convection zone has poleward Lorentz force (poleward slip tendency)

This opposes meridional circulation there

(Karak & Choudhuri, in preparation)



Conclusions

- Grand minima are caused by combined fluctuations in poloidal field generation and meridional circulation
- Small scale dynamo must be operating besides the global dynamo creating small scale magnetic fields
- Hydrodynamics of differential rotation and meridional circulation is a challenging problem to study through either DNS or mean field theory
- It is more manageable to study the modifications of differential rotation and meridional circulation due to the Lorentz force of dynamo-generated magnetic field

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